Common Core Math: A Guide

Stephanie J. Eaton
Western Oregon University

Follow this and additional works at: https://digitalcommons.wou.edu/honors_theses

Part of the Curriculum and Instruction Commons, and the Science and Mathematics Education Commons

Recommended Citation
https://digitalcommons.wou.edu/honors_theses/5

This Undergraduate Honors Thesis/Project is brought to you for free and open access by the Student Scholarship at Digital Commons@WOU. It has been accepted for inclusion in Honors Senior Theses/Projects by an authorized administrator of Digital Commons@WOU. For more information, please contact digitalcommons@wou.edu, kundas@mail.wou.edu, bakersc@mail.wou.edu.
Common Core Math: A Guide

By

Stephanie J. Eaton

An Honors Thesis Submitted in Partial Fulfillment of the Requirements for Graduation from the Western Oregon University Honors Program

Dr. Breeann Flesch,
Thesis Advisor

Dr. Gavin Keulks,
Honors Program Director

Western Oregon University

June 2014
Table of Contents

Part I
History ................................................................. 4
Standards for Mathematical Practice ................................. 18
Interview Commentary ............................................... 29
Suggestions for Future Study ................................... 39

Part II
Website Screenshots ................................................ 42

Appendix
Interview Questions ................................................. 103
Sticking Points Progression .................................... 105
References ............................................................. 132
Part I
History

The state of Oregon has been realigning, reformatting, essentially re-creating its educational math standards every few years for over a decade. For today’s parents, this not only means that an entirely new system is being used to teach their children math than when they went to school themselves, but that their children are learning a completely new way of looking and dealing with math altogether. The progression of alterations made to standards has led to a more content-oriented curriculum, which aims toward deeper student understanding of the same math concepts taught years ago. Current standards point their focus away from rote memorization of integer operation facts (e.g. times tables), and toward mastery comprehension of the “why” and “how” of mathematical reasoning. Students are being taught to become mathematical thinkers rather than mechanized reiterators.

According to the Oregon Department of Education (ODE), academic content standards are “statements of what students are expected to know in particular subjects and be able to do at specified grade levels” (ODE, 2010, p.1). These expectations provide teachers with a general guideline to which they orient their lessons throughout the year, and which they use to assess student learning.

2002 No Child Left Behind

In 1995, due to new legislation, education in Oregon began transitioning from primarily skill-oriented standards to standards which were more content-oriented. The
content was to be based off of academic content standards, which provided teachers across the state with a common understanding of expectations for student learning in specific content areas. Students were tested for their understanding of these academic content standards in grades three, five, eight, and 10, and were assessed using a set of achievement standards aligned to the content standards (ODE, 2010).

Proposed in 2001, the No Child Left Behind (NCLB) Act was signed into law January of 2002 (United States Senate, n.d.). With the arrival of NCLB, student testing was extended from the four-grade assessment schedule, and standards were further realigned to be increasingly content-oriented. The NCLB Act was originally proposed due to reports of low student achievement in several academic areas - though specifically in reading - despite exponential growth in elementary and secondary educational funding (U.S. Department of Education). It was hoped that the law would build teacher and school accountability for received funding, and that such accountability would raise student achievement.

NCLB was primarily focused on raising reading ability levels, but also had a strong focus on regularly assessing student progress. According to the U.S. Department of Education in their parents guide to the No Child Left Behind Act, in order to assess student knowledge and understanding in math, yearly tests were administered in grades three through eight and once in high school (2003). Results of school, district, state, and national tests were made available to the public, and parents were additionally given their individual student’s private scores. Test results were meant to do four main things,
referred to as “common sense pillars”: build accountability among teachers and school officials for student learning; expand parental options and involvement; “expand local control and flexibility”; and create an emphasis on teaching using methods based on “scientific research” (which the document later explains are methods that have reliable evidence of effectiveness) (U.S. Department of Education). School performance under NCLB was evaluated based on its student test scores, and - if scores were collectively too low – the district was given a certain amount of time to raise scores and meet the Adequate Yearly Progress standard set by the government before corrective actions were taken — actions which primarily involved a decrease in federal funding.

2006 ODE’s Contract with WestEd

In 2006, the Oregon Department of Education began its first large-scale review since 2000 of its education curriculum and standards (State of Oregon, 2007). This review was prompted by the publication “Curriculum Focal Points” from the NCTM in 2006, which placed heavy focus on arithmetic, geometry/measurement, and algebra in order to organize the grade-progression math concept trajectory (NCTM, Focal Points). It was hoped that the focal points would bring coherence to math education across the grades by connecting concepts “logically with the mathematics in earlier and later grade levels” (NCTM, Focal Points).

ODE contracted with WestEd, a non-profit based in San Francisco to examine its standards as well as answer a list of “critical questions” (State Board of Education, 2007).
Several sets of these questions as they were addressed were as follows (Beckmann, 2007):

- What is an appropriate structure for the content standards?
- What are the advantages of adopting core standards?
- What is essential content?
- What is an appropriate review and revision schedule for content standards?
- What is the appropriate relationship between content standards and high school diploma requirements?
- How should high school standards be restructured to accommodate the new diploma requirements?
- How might Oregon’s assessment system change with the standards to better guide instruction and learning?
- What should Oregon’s policy be for providing assessments in alternate languages?
- How do we support special populations in framing the content standards and assessing mastery of content?
- How will testing accommodations be affected by changes to the standards and assessments?
- What are the trade-offs between time for instruction and reliability of the assessment?
- How were survey results used in responding to the Critical Questions?

These questions were designed to seek advice on the physical organizational structure, as well as on the appropriateness (by grade and by population) of what would be the new standards and assessments. The questions on appropriateness and content made the State of Oregon subject to the possibility of content shifting among grades; redefining what content was “essential” and “appropriate” at which grades would completely reformat the trajectory through the grades. WestEd responded to these questions in the form of recommendations. Mirroring the ODE questions, the WestEd recommendations focused on the organization of the standards and how students would progress through them (i.e.,
the trajectory). A partial list of general WestEd recommendation topics as presented to the State Board of Education on May 18, 2007 is as follows (State Board of Education, 2007):

- The structural format of Oregon’s academic content standards
- The rigor, clarity, and breadth of content standards in each subject
- Core standards and essential skills
- The valid uses of the Oregon Statewide Assessment System
- Alignment of K – CIM content standards and assessments
- The vertical articulation of K – CIM standards and their alignment with post-secondary standards and assessments

2007, August Results of WestEd Contract

On August 31, 2007, WestEd released a final report on its evaluation of Oregon standards, having based their assessment of (and recommendations for) Oregon’s math standards on Indiana’s math standards as well as two National Council of Teachers of Mathematics (NCTM) publications: “Principles and Standards for School Mathematics” and “Curriculum Focal Points for K-8 Mathematics” (WestEd, 2007). Their findings state that Oregon’s math standards are “of good quality and reflect a breadth and depth of content coverage… are clearly written, [and] focus on important skills/concepts for instruction” (WestEd, 2007). The report also contained a list of general recommendations for guiding ODE through revising their current grade-level standards. The general recommendations were:
• **Evaluate standards for redundancy within a grade**: this recommendation emphasizes the need for efficiency in the new standards. Eliminating redundancy would narrow teaching focuses and create a smoother progression through the concepts for students.

• **Evaluate standards for level of detail - some standards have elaborate detail and others do not**: this recommendation emphasizes the need for consistency in the new standards. Consistency in the amount of detail given for each standard would help teachers to better interpret all of the standards, rather than the few elaborately detailed ones. This would also create a consistency across the state in how teachers interpreted the new standards.

• **Differentiate between detail that describes the curriculum to be taught and the intended learning outcomes (content skills, outcomes)**: this recommendation emphasizes the need to more clearly define the intention of each standard. Standards define one of two things or both: what teachers should be teaching, and/or what students should be learning. These two things are not always identical, and are sometimes very ambiguously connected. Clarifying the intentions of the standards would help focus teacher and student attention on the areas which the State deems are most important.

• **Evaluate coherence across descriptive statements for a given standard... in terms of appropriateness of degree of specificity (i.e., from general/broad to specific) and clarity of relationships**: this recommendation emphasizes the need to de-generalize the standards into specific and clearly defined statements. It also emphasizes the need
to clarify how the standards are interrelated. De-generalizing would again help clarify
how teachers should interpret the standards, and clarifying the interrelationships
between the standards would help focus the content trajectory.

- **Verify that expansion/extension of content reflected in standards is reasonable and
  intentional:** this recommendation emphasizes the need to rationalize the detail of
  content in the new standards to which teachers hold their students accountable. This
  rationalization would help keep teachers focused on the intended student goals and
  outcomes of the standards, rather than the specific content used to lead students to
  those goals and outcomes.

- **Evaluate the clarity and effectiveness of small variations in the wording/language of
  specific standards across the grade levels - verify that the differences are intentional
  and reflect clear, real differences in expectations of students vis-a-vis the content/
  skill:** this recommendation emphasizes the need to determine a consistent language to
  be used in the new standards. Differences in language, no matter how small, can lead
to differences in teacher interpretation of the standards across the grade levels, which
can lead to differences in teacher expectations from one grade to the next. Developing
a consistent language to be used in the new standards would help teachers in reliably
and predictably interpreting the standards, which would in turn ease student content
progression from one grade to the next.

- **Embed definitions of key content terminology, if interpretations of terminology can be
  varied AND the intent of the state is to promote consistency:** this recommendation
again emphasizes the need for consistency in the language of the standards.

Developing an agreed-upon content vocabulary which is consistent across the state would help to create a single, unbiased, state-wide assessment which could be relied upon to generate dependable evaluations of student knowledge and achievement.

• *Make sure the language in the standards clearly conveys expected skills and complexity:* this recommendation emphasizes the need for clarity in the language of the standards. Clarifying language used within the standards would help decrease the variations of interpretations made by teachers across the state, which would help create consistency in both student exposure to and understanding of the content intended by the standards.

• *Evaluate coherence and consistency of content emphases across grades:* this recommendation generalizes the collective essence of the previous recommendations. Creating coherent standards would help teachers implement the standards in the ways in which they were intended, and creating consistent standards would help give students state-wide an equitable exposure to the content. Making the content standards both coherent and consistent across the grades would allow for a smoother, more efficient content trajectory for students to progress through.

An overall theme came out of the WestEd recommendations, and it articulated similarly to the ideas found in NCTM’s “Curriculum Focal Points”: ODE needed to create standards that had clarity, consistency, and coherence across the grades. Soon after WestEd’s final report was released, Oregon adopted a new set of math standards.
2007 Content Standards

The first version of Oregon’s content standards was drafted in January 2007, and the final version was drafted the September after WestEd’s final report was released (ODE, 2007). The Oregon State Board of Education officially adopted the final version a few months later, which attempted to reflect WestEd’s evaluation and recommendations.

A goal of these new standards was to make a shift from what had come to be known as the “mile wide and inch deep” curriculum and standards of the 2002 standards under No Child Left Behind. The 2007 content standards offered students and teachers a more manageable range of standards, which in turn allowed for greater depth and understanding. The state also felt the need to clarify the path which students were expected to follow as they moved through the grades, so these content standards also offered students and teachers a more organized trajectory, of sorts, which more clearly showed expected student progression through the math curriculum by grade.

The new standards document was organized first by grade level, then by core standard. Each grade was partitioned into three course standards, which were to be the grade’s three primary areas of focus. Each course standard contained a set of more detailed content standards that together made up the grade’s complete list of math standards. These lists turned out to be far shorter than those from the previous 2002 standards, which often repeated standards from grade to grade.

The content standards were created and organized with five mathematical skills in mind, known as the NCTM Process Standards: Problem Solving, Reasoning and Proof,
Communication, Connections, and Representations (NCTM, *Process Standards*).

Students were meant to develop these skills as they progressed through the grades in the hopes that the skills would positively affect their success in both math and other content areas.

**2008 Comparing Standards under NCLB with new 2007 Content Standards**

As part of their contract with ODE, WestEd prepared a document known as the “Crosswalk”, in which it presented a comparison of Oregon’s recently adopted 2007 math content standards, which had taken the place of the 2002 math Grade level standards that were adopted under No Child Left Behind. The Crosswalk document was submitted in two parts on May 22, 2008. Part A of the Crosswalk showed how each 2002 math standard lined up with one or more of the new 2007 math content standards, which allowed educators to search for standards in the 2007 set by looking up a known 2002 standard. Part B of the Crosswalk showed how each of the new 2007 math content standards lined up with one or more of the 2002 grade level math standards, which allowed educators to refer back to a 2002 standard that could help clarify a new known 2007 standard. Together, Crosswalk A and Crosswalk B presented comparisons of the 2002 and 2007 math standards in a way that allowed educators to more easily make note of the many changes that were made to the math standards and curriculum by referring back and forth between the two.
In addition to connecting each 2002 standard to one or more 2007 content standards, Crosswalk A indicates whether the 2002 standard matches above, at, or below grade level with the corresponding 2007 standard(s), and whether the 2002 standard matches above, at, or below the complexity level of the corresponding 2007 standard(s).

The primary difference between the 2002 and 2007 standards is the actual number of topics covered; the 2007 content standards provide a drastically reduced list of topics (by nearly 50%) compared to that of the 2002 grade level standards (ODE, 2008). Less noticeable is the shift in focus in grades one through five onto arithmetic, and in grades six through eight onto preparation for algebra. The 2007 standards, already being far less numerous, were also written to be both more precise and more flexible than their 2002 counterparts. Overall, the change from the 2002 to 2007 standards was marked in the new organization and progression of standards from one grade to the next.

**NCLB Waiver for Oregon (2012)**

In July 2012, Oregon was officially granted a waiver from the requirements of No Child Left Behind as a result of having adopted the new Common Core State Standards, joining 32 other states who had been granted waivers or had waivers in progress (ODE, 2012). Whereas NCLB required schools and districts to be evaluated based on student assessment scores, Oregon’s waiver plans emphasize a school evaluation system of accountability and support (ODE, 2012). The Adequate Yearly Progress evaluation under NCLB will be replaced with a new school report card which
focuses on (ODE, “Comparing NCLB”): individual student growth; graduation and subgroup graduation goals; and authentic college and career readiness measures. Another major shift away from NCLB requirements is the way in which the state will identify and intervene in and support its lowest performing schools (ODE, “Comparing NCLB”): the bottom 15% of Title 1 schools will be termed either “priority” or “focus” schools. These schools and their districts will be required to develop individual Comprehensive Achievement Plans in which support and intervention steps will be outlined (ODE, “Comparing NCLB”). Examples of such steps include implementing new instructional methods or materials, extending the school day and/or week, improving communication between teachers and families, changing leadership role-holders, and providing tutoring/etc. services to individual students.

This change in policy is reflected in the Standards for Mathematical Practice of the new Oregon Common Core math standards, which promote a change in focus from “passing the test” to “understanding the concepts”.

**CCSS (2010 - present)**

In October 2010, Oregon officially adopted new math standards, created as part of the Common Core Standards Initiative by the National Governor’s Association Center for Best Practices and the Council of Chief State School Officers (Common Core State Standards Initiative, 2011). Oregon plans to fully implement the Common Core State Standards for Math (Math CCSS) by the 2014-15 school year (ODE, 2010). The goal of
the Math CCSS was primarily to create a focused, coherent set of national standards that promotes consistency across the states, as well as progress towards national development in math skill and understanding (Common Core State Standards Oregon).

In an attempt to bridge the gap between test- and understanding-focused standards, the Math CCSS were developed around eight Standards for Mathematical Practice (Common Core State Standards Oregon):

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attended to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

These eight mathematical practices have been popularly structured into four themed groupings, which were meant to further help teachers implement the standards in the classroom (McCallum, n.d):

- 1, 6 – “overarching habits of mind of a productive mathematical thinker”
- 2, 3 – “reasoning and explaining”
- 4, 5 – “modeling using tools”
- 7, 8 – “seeing structure and generalizing”

The Math CCSS are grade-level based, and introduce each grade’s standards by stating its two to four individual focus areas (Common Core State Standards Oregon). These focus areas are overarching statements of what students should learn in each grade; they set up the basic topics or skill areas around which the standards are based for each grade.
(Common Core State Standards Oregon). Each of the elementary grades (K-5) has been broken into four to five domains, which serve as headers for sectioning off the individual standards (Common Core State Standards Oregon):

- Counting and Cardinality .......................... CC ..... (K)
- Operations and Algebraic Thinking .......... OA ..... (K-5)
- Number and Operations in Base Ten .......... NBT .... (K-5)
- Measurement and Data ............................ MD ..... (K-5)
- Geometry ................................................. G ....... (K-5)
- Number and Operations - Fractions .......... NF ..... (3-5)
Standards for Mathematical Practice

The Standards for Mathematical Practice are the non-content practices within the Common Core Standards. There are a total of eight practices, which are often divided into four groupings of two practices each. These groupings were created by William (Bill) McCallum, a professor at the University of Arizona, and can be categorically generalized as groupings of thinking, reasoning/explaining, modeling, and recognizing (McCallum). Breaking the eight practices into these four groupings further emphasizes their intended foci, making them easier for teachers to interpret and translate into classroom use.

The role of the practices in the classroom can arguably be seen as goals for achieving mathematical automaticity (the ability to do a mathematical act accurately with little to no thinking). Teaching automaticity within the math content area is not a new undertaking; rote memorization of facts (primarily multiplication, but also addition to 5 and to 10 in the younger grades) has been a popular method of generating math automaticity (Woodward, 2006). The idea has been that memorizing the multiplication facts will lead to quicker and more efficient processing of “estimation, mental calculation, and approximation skills… [which] are part of the development of what has been often referred to as ‘number sense’” (Woodward, 2006). These new standards for math practice offer a means of providing a framework for building automaticity in math
— which can be used alongside rote fact memorization — by generating a paradigm shift in which students approach and interact with math on a more eager, introspective level.

**Overarching Habits of Mind of a Productive Mathematical Thinker**

The first of the four groupings contains practices which McCallum describes as “overarching habits of mind of a productive mathematical thinker” (McCallum). These two “thinking” practices are Standards for Mathematical Practice numbers one and six (MP1 and MP6), which are “Make sense of problems and persevere in solving them” and “Attend to precision”, respectively (Common Core State Standards Oregon).

**MP1 — Make Sense of Problems and Persevere in Solving them**

MP1 is made up of two core parts: making sense of the given problem, and persevering in solving that problem. In order to “make sense” of a given math problem, more skills are needed than are generally taught in the standard math class. There are several steps required in making sense of a problem, beginning with arguably the most important: being able to read the problem. Once the words on the page can be read, students must place meaning to each and, in turn, make connections from one word to the next. This often means students must be able to infer information not explicitly stated in the text of a problem. These literacy skills — including the ability to infer meaning and implicit details in the text — are critical to making sense of the problem because they allow students to understand and make connections with the problem’s context.
The second half of MP1 is perseverance, which Merriam-Webster defines as “continued effort to do or achieve something despite difficulties, failure, or opposition”. Within any of the academic content areas, students are bound to be confronted with challenges on a daily basis. A healthy amount of perseverance will push students to continue trying until they have reached the problem’s solution or have made a breakthrough in the process. As put by Thomas R. Hoerr in his article *Good Failures*, “all students need [the ability to persevere] because sooner or later they will fail at something. Their success will depend on their ability to hang in, bounce back, try and try again, and persevere” (84). Students encounter failure sooner or later in math, but having that ability to persevere will aid them in overcoming those failures.

**MP6 — Attend to Precision**

MP6, the second “thinking” standard, emphasizes the importance of giving attention to both big and small details in mathematics, specifically in word problem formatting and procedural (non-word problems/equations/expressions) mathematics. The “big” details in a math problem help to set up the question and present some form of general context, while the “small” details help to create and follow the required mathematical path to the solution. Thus when students are presented with a problem, they are being asked to simultaneously attend to the details of the question, the context, and the procedural math itself.
Attending to the problem’s question directs student focus at looking ahead to how the answer should be formatted, and helps students decide what kind of information they will need in order to find the answer. The problem context generally includes details related to how and why the question is being asked; while word problems provide real-life setting, actions, and outside (often implicit) forces, procedural math problems remain explicitly numerical and symbolic, and require knowledge of equation/expression formatting. Finally, attending to the steps and procedures of the procedural math itself requires precision by following the rules of mathematics as determined by number theory (e.g., \(2+2\) in base ten will always equal \(4\) in base ten). Attending to the precision — learning to be precise — in these procedural calculations is the focus of primary-level mathematics instruction, beginning with addition facts.

Though the details define the problem and make it solvable, they can also over-complicate a problem when too many unnecessary or redundant details are presented. Without attending to the question details, an overabundance of details within the problem context may steer students toward finding solutions to questions which have not been asked. Without attending to the problem context details, an overabundance of context details may cause students to misuse the numerical components, ending in a non-sensical problem solution.
Reasoning and Explaining

The second of McCallum’s four groupings contains Standards for Mathematical Practice numbers two and three, which he describes as the “Reasoning and Explaining” practices. These practices are “Reason abstractly and quantitatively” (MP2) and “Construct viable arguments and critique the reasoning of others” (MP3) (Common Core State Standards Oregon).

MP2 — Reason Abstractly and Quantitatively

MP2 focuses on students’ mathematical reasoning, primarily their abilities to decontextualize and to contextualize information as ways of “creating a coherent representation of the problem” (CCSSO). Decontextualizing a problem involves stripping a problem of its real-life context and representing it using numbers and symbols; for example, decontextualizing “two red apples and two green apples are four apples total” into $2+2=4$. To contextualize a problem is to take a numeric-symbolic problem and represent it as words with a real-life context; for example, contextualizing $2+2=4$ as “two red apples and two green apples are four apples total”.

Students are asked to decontextualize a problem more often than they are asked to contextualize a problem, as contextualizing a problem requires more higher order thinking skills, as defined by Bloom’s Taxonomy (Overbaugh). Whereas the ability to decontextualize problems is more likely to fall under the Understanding and Analyzing
areas of the taxonomy, the ability to contextualize problems is more akin to components of the Evaluating and Creating areas of the taxonomy.

**MP3 — Construct Viable Arguments and Critique the Reasoning of Others**

MP3 involves two core parts: constructing an argument, and critiquing an argument; it is the “Explaining” standard. The “argument” being referred to is the interpretation of, the solution to, and the reasoning behind a solution to a given problem. These arguments are made based on the available data, as well as facts, procedures, and definitions.

Students construct formal as well as informal arguments by “build[ing] a logical progression of statements… analyz[ing] situations… recogniz[ing] and use[ing] counterexamples… [and] mak[ing] plausible arguments that take into account the context” (CCSSO). Students are thus taught how to think logically through a problem beginning at the first step and making their way progressively to the solution, where each step builds upon the last. Understanding the process of constructing an argument is necessary to critiquing the reasoning of others. Once students are aware of how to create the argument, they can better see flaws, omissions, and other inefficiencies/inaccuracies within the reasoning of others.
Modeling and Using Tools

McCallum’s third grouping, which he notes as the “Modeling and Using Tools” practices, contains Standards for Mathematical Practice numbers four and five. These practices are “Model with mathematics” and “Use appropriate tools strategically”, respectively (CCSSO).

MP4 — Model with Mathematics

Modeling mathematics involves physical models (i.e., three-dimensional constructions), as well as models in the form of drawings, tables, graphs, contextualized word problems, and decontextualized numeric/symbolic representations (i.e., equations and expressions). MP4 emphasizes the importance of student ability to use these mathematical models to solve everyday-type contextual problems. In this sense, modeling involves simplifying — decontextualizing — situations through being able to distinguish the important from the unnecessary information, as well as using visual tools (for example, a graph) to represent the simplified situation and “draw conclusions” (CCSSO).

This standard further emphasizes the importance of placing the decontextualized answer back into the context of the situation in order to make sure their answer makes sense within the given situation. Students are often asked to draw conclusions from models — especially graphs and tables — and thus must be able to understand how each model type can be interpreted. In this manner, modeling requires students to be able to contextualize in addition to being able to decontextualize as the situation warrants. To
interpret a model, students must know how the model is constructed, how the labels affect the model, and — in many cases — it proves beneficial to be able to recognize the model’s various forms. Students are often introduced to interpreting the different types of models through decontextualizing a problem to create the model from scratch.

**MP5 — Use Appropriate Tools Strategically**

MP5 is made up of two core parts: using appropriate tools, and using those tools strategically. The tools being referred to are not only physical (e.g., calculator, protractor, graph paper, etc.), but also include mental tools: procedural strategies used to get from concept A to concept B. The shift away from “kill and drill” rote memorization is one of the largest paradigm shifts within the new standards, with the new focus being on these mental tools, these procedural strategies which are intended to leave students with a deeper understanding of the ‘how’ and ‘why’ rather than the mere answer.

The term “appropriate tools”, is not meant to recognize a certain set of tools as being the only tools appropriate for classroom use, but instead is used to recognize the fact that some tools are more appropriate than others for any given context. A real-life example might be students learning when to use a ruler versus a measuring cup, and why one is more appropriate than the other. To use tools “strategically”, students must first understand which tools would be most appropriate for the given situation, then must decide how best to implement those tools. These decisions must often be made between
efficiency and precision (versus estimation), and in order to effectively make these decisions, students must have an understanding of the tools they have at their disposal.

Rote memorization of basic math facts could also be considered a tool which — in an appropriate situation — becomes more of an advantage than other available tools; generally, these situations would be those which require efficiency and time-management. Memorization of facts decreases the space needed in the short-term memory (the limited capacity) to solve a problem, which allows students to more easily move through and understand a problem, especially multi-step problems.

**Seeing Structure and Generalizing**

The final of McCallum’s four groupings, which he notes as the “Seeing Structure and Generalizing” practices, contains Standards for Mathematical Practice numbers seven and eight. These practices are “Look For and Make Use of Structure” and “Look for the Express Regularity in Repeated Reasoning”, respectively (CCSSO).

**MP7 — Look for and Make Use of Structure**

The structure referred to in MP7 includes structures such as patterns, rules, and problem format archetypes (i.e., the general use of the term “structure” in a physical sense). Looking for these structures involves students learning pattern recognition strategies in early grades, then building upon those strategies in the later grades. MP7 specifically states that students should be able to “step back for an overview and shift
perspective” (CCSSO, 8). By this, it is meant that students should be able to look at a new and/or difficult/complex problem from a (physical) structural view and reason about a pattern/rule-based structure which is inherent or acquired within the problem.

Once students have looked for and found a given structure, they must then use the structure to solve the problem. This is where having the pattern recognition strategies from the early grades readily on deck (in the long-term, unlimited capacity memory) becomes practical and highly beneficial. Once a structure is found, students must understand that particular structure — the patterns and rules behind it — in order to effectively use it. An early understanding of patterns and rules, how they look and what they do, is the foundation necessary to understanding structures in later grades.

**MP8 — Look for and Express Regularity in Repeated Reasoning**

MP8, the final practice standard, focuses on student ability to recognize and effectively make use of repeated reasoning, or, “shortcuts” (CCSSO, 8). This repeated reasoning, these “shortcuts”, are similar to the structure from MP7, but fall under the specific pattern structure of repetition. Though repetition is itself the foundation of most patterns, the repetition here is done in the *reasoning*, or the thinking behind a solution. MP8 notes that students should be able to find this repetition by “noticing the regularity” and “maintain[ing] oversight of the process” while solving problems (CCSSO, 8). This means students must be able to keep the whole problem in their view rather than focusing all of their attention on pieces of it.
These eight practices together create a foundation for mathematical automaticity; they are the non-content practices which make the content standards achievable. Before the Common Core, these practices were expected from students, though not explicitly taught or required by the standards. Now, students will be held to these practices, and instruction will be based around teaching content through them.
Interview Commentary

The following commentary was based on two separate group interviews, consisting of questions regarding the new Common Core Math standards. Interviewees were read each standard in turn, and were asked whether the content had/has been a consistent area of difficulty for students in their respective grades. Interviewees were also asked to speculate on the causes of the difficulties, and were asked to generate a list of topics which they would like to see practiced and/or addressed more in the students’ homes.

The interviews were conducted by grade and consisted of two grade-specific group interviews. The fourth grade group was made up of three fourth-grade teachers (Teachers A, B, and C), and the fifth grade group was made up of two fifth-grade teachers (Teachers D and E). All five interviewees had classrooms which were influenced by the surrounding low-income/high-poverty area; the school was a Title 1, with 100% free/reduced lunch. The teachers interviewed reported being in the profession between 8 and 40 years. Each self-identified as having some level of familiarity with the new Common Core Math standards which was anywhere from six to eight out of ten, with seven out of ten being the most common familiarity ranking.

Each of the group interviews resulted in a list of its own grade-specific “Sticking Points” (a phrase used here to refer to areas of study which have consistently been shown to cause students a level of difficulty). The two lists, created by teacher responses to each
of the respective grades’ Common Core Math standards, also contained a set of Sticking Points which were common across the two grades. The resulting Sticking Points are arranged here by those unique to grade 4, those unique to grade 5, and those common to both grades, with an ending section of teacher requests.

Grade 4 Sticking Points

The fourth grade interviews resulted in five Sticking Points which were not common to the fifth grade results: three from the Operations and Algebraic Thinking (OA) domain, one from the Numbers and Operations in Base Ten (NBT) domain, and one from the Number and Operations — Fractions (NF) domain.

The first Sticking Point from the OA domain was the ability to solve both single- and multi-step word problems (standards 4.OA.2 and 4.OA.3). The group’s consensus was that students lacked the ability to visualize the problem: Teacher C noted, “They can’t visualize [the problem]… The kids who can visualize it can do [the problem]. The kids who cannot, whether it’s one or five steps, they struggle”. The second Sticking Point from the OA domain was the ability to assess the reasonableness of an answer (4.OA.3), though Teacher B observed that the difficulty may lie in remembering or thinking it necessary enough to make the assessment: “Once [the students] are pleased with the process, they go for it, and that seems to be the end of their analysis”. The final Sticking Point from the fourth grade OA domain was the ability to find factor pairs (4.OA.4).
Specifically, Teacher A noted that students have less difficulty with finding factor pairs of numbers “up to… 24, 30, [but] when it gets larger, they leave one pair of factors out”.

The Sticking Point from the NBT domain was the standard for subtracting multi-digit whole numbers with borrowing (4.NBT.4). The standard specifies the use of the standard algorithm, though the agreement within the group was not that students found difficulty in the standard algorithm; rather, that students show carelessness in the problem’s written organization, which causes mistakes to be made merely because students “can’t read it” (Teacher B). This lack of organization in subtraction was described as a lack of attention to keeping digits in a straight vertical line (physically), which leads to digits being placed and operated within the incorrect place value position.

The final Sticking Point specifically related to grade four, from the NF domain, was the ability to compare fractions with different denominators (4.NF.2). Comparisons require students to either change the format of the fractions being compared (i.e., to equivalent fractions with like denominators) or compare the fractions with benchmark fractions, both of which Teacher C observed as being difficult for students: “if [students] have to change [the fraction] to the like denominator or use the landmark fractions, they really struggle with it and they give up. That’s what I find. It’s that persistence and perseverance”.
**Grade 5 Sticking Points**

The fifth grade interviews resulted in seven Sticking Points which were not common to the fourth grade results: one from the Operations and Algebraic Thinking (OA) domain, two from the Number and Operations in Base Ten (NBT) domain, one from the Number and Operations — Fractions (NF) domain, one from the Measurement and Data (MD) domain, and two from the Geometry (G) domain.

The first Sticking Point specifically related to grade five, from the OA domain, was CCSS 5.OA.1, which focuses on the order of operations. Even with the recognized acronym PEMDAS (Parentheses, Exponents, Multiplication/Division, and Addition/Subtraction) and the many phrases used to remember the acronym at their disposal (e.g., “Please Excuse My Dear Aunt Sally”), Teacher D noted that students still have difficulty remember the order: “[students] didn’t know whether to do the addition, multiplication, or subtraction first”. Teacher D also observed that even when students correctly identify and employ the correct procedural order, if students make a single computational error, it “throws off [their] whole answer”.

The first of the two Sticking Points from the NBT domain was the ability to multiply by powers of ten (5.NBT.2). Teacher E mentioned that students “didn’t know whether to move the decimal [to the left] or [to the right]”. The consensus of the group was that this Sticking Point’s difficulty originated from a confusion in procedure which was due to students not yet having developed a firm understanding of place value. The second Sticking Point from the NBT domain was also related to place value: the ability to
compare decimals (5.NBT.3a/b). Teachers D and E both agreed that students tend to get “tunnel vision”, where their focus is only on the digits to the right of the decimal, forgetting the digits to the left. Teacher D noted that students don’t “look at the whole number. If the whole number is larger, that’s [a] dead give-away”. For those numbers with equivalent whole numbers to the left of the decimal, the problem then becomes “a matter of making sure [students] had an equal amount of digits to compare” (Teacher D).

The fourth sticking point, from the NF domain, was adding and subtracting fractions with unlike denominators (5.NF.1). This standard includes adding and subtracting improper fractions, and specifies that students should first change all fractions involved into equivalent fractions, which Teacher E observed was a possible result of non-mastery in basic multiplication facts (this will be discussed later in this paper).

From the MD domain, the fifth sticking point was finding the volume of solid figures (5.MD.3b and 5.MD.4). Teacher E specified that the difficulty was in finding the volume of irregular figures, and that students “seem to catch on with basic solid figures”. The group agreed that students had difficulty in partitioning an irregular figure into several separate regular figures.

Of the final two fifth grade Sticking Points from the G domain, the first was answering questions by graphing within the four-quadrant coordinate plane (5.G.2). This standard was discussed in two parts: graphing skills and word problems. According to the group, the students generally lacked the skills to graph in any quadrant other than the first, which Teacher D observed might be due to a struggle with vocabulary. As for word
problems, Teacher E noted that students “don’t pay attention to a lot of detail… they just want to read [the problem] once and be done. So it’s a matter of being careful”. The final fifth grade Sticking Point was the ability to name shapes and figures based on given attributes (5.G.3). Though the group agreed that students were able to identify a figure’s attributes when shown the figure in question, students had difficulty with knowing what figure was being described when given a list of attributes.

**Sticking Points Common to Both Grades**

The two grade-level lists of Sticking Points based on the group interviews included a set of four common points: two from the NBT domain, one from the MD domain, and one which was created from a combination of the fourth grade OA and fifth grade NF domains. Though other Sticking Points could be said to be similar across the grades, these four had the most direct commonalities.

The first common Sticking Point from the NBT domain was long division (4.NBT.6 and 5.NBT.6). The fourth grade standard involves single-digit numbers being divided into up to four-digit numbers and includes remainders, whereas the fifth grade standard involves double-digit numbers being divided evenly into up to four-digit numbers. Both groups agreed that the reason for the difficulty lies in division’s connection to multiplication; Teacher C explained that students “who don’t have their multiplication facts down… can’t do it”. In addition to lacking the necessary
multiplication skills, Teacher A noted that there is little long-term retention: “if there’s too much [time]… since we do one, then it has to be refreshed again”.

The second common Sticking Point from the NBT domain involved rounding (4.NBT.3 and 5.NBT.4). The grade four standard mentions only multi-digit whole numbers, whereas the grade five standard primarily deals with numbers that have digits to the right of the decimal. Both groups agree that rounding is generally “a tough one” (Teacher E), though Teacher C notes that students begin to pick up the skill “after lots of repetition”. Both groups also agree that they have found many students respond positively to memory tricks such as the rhyme given by Teacher C: “four or less, let it rest; five or more, raise the score”.

From the MD domain, the third common Sticking Point was converting between units (4.MD.2 and 5.MD.1). There is little difference between the two grades’ standards, and for the most part, the groups were in agreement that the reason was a general lack of student exposure to the involved units. For example, Teacher E commented “if I said to my class: ‘What’s a centimeter?’ they still would [ask] ‘Is that the big one or is that the little one or is that the really little one?’ because we don’t use it on a daily basis”.

The fourth common Sticking Point, from the combined fourth grade OA and fifth grade NF domains, was the ability to know which operation to use in word problems (4.OA.2 and 5.NF.2). There was no general consensus of the two groups as to the cause of the difficulty. Teacher C referenced student inability to visualize, Teacher A referenced the general lack of attention to detail, and Teacher D mentioned student unwillingness to
follow multi-step problems through to completion (“[students are] accustomed to one step, so they want to be done with it after one step”).

There was an additional Sticking Point which the grade levels had in common, but which was neither standard- nor domain-specific: basic math facts — specifically addition and multiplication. Of the many Sticking Points, the idea of basic math facts as being a problem area was mentioned most often in both groups. The standards in which a deficiency in basic fact fluency was noted include: 4.OA.4 — “they don’t know their multiplication tables [enough] to be able to do factors” (Teacher C); 4.NBT.6 — “[the students] who don’t have their multiplication facts down can’t do it” (Teacher C); and 5.NF.1 — “if you can’t multiply and divide, you really are at a loss” (Teacher E).

Requests for Help from Home

In addition to the questions regarding each of the standards, the interviewees were asked for a list of items from the standards they wished to receive help for from students’ parents/guardians at home. A total of four topics were raised: three which were noted by both groups, and one which the fifth grade group added further.

In both grades, all teachers (A—E) requested that students be held accountable for learning their basic math facts to mastery. Teacher B stated that “rote recognition… would be a huge time saver, and [the student] would be so much more successful”. The fifth grade group requested that students practice addition and multiplication facts at home, enough to have “instant recall” of the basic math facts (Teacher E). Both groups
noted that memorization of the basic math facts are not a part of their standards under Common Core, but that the “instant recall” would aid students in many areas which are a part of the Common Core standards.

Three primary skills were observed by both groups to be topics which would best be practiced and mastered at home once introduced at school in the earlier grades: telling time, counting/exchanging money, and measuring with a ruler. None of the three skills are part of the standards for grades four or five, but the teachers noted that time is still spent on “catching students up”. Teacher C observed: “getting [students] to where they always get the clocks, they always are able to add up the coins… I feel that that would help because it would help with our decimals when we do the money… and being able to use a clock and tell time”.

The third topic raised by both groups was learning language through conversation. Teacher B noted that students are “missing their shared language experience”, which Teacher C added might be a result of the culture in the community. The fifth grade group requested that parents take the time to generate student respect for both language and math through mathematical conversations at home, an example being fractions and measurement through measuring cups while cooking, which the fourth grade group suggested. Teacher C also suggested that family conversation “develops oral language, [and] develops concepts that [students] are missing”.

The fourth topic, raised by Teachers D and E, was student accountability for homework. The group agreed that they needed support from parents to keep the students
themselves accountable for completing and returning their homework. Teacher E expressed that when parents take the time to show their concern for student homework, the student sees “that the parents feel that education is important. But if [parents] never look at their [student’s] work, the kids get the message that the parents don’t care”.
Suggestions for Future Study

One area of possible future study would be to continue the research of Sticking Points down into the younger grades. As mathematics education is taught through a progression of interconnected skills, knowledge, and abilities, early problem areas have the potential to cause later problem areas to “stack up”. Researching early-grade Sticking Points would offer more insight into where problems begin, allowing parents and teachers to target the appropriate skills/abilities to better anticipate and prevent students from falling behind.

Another area of possible future study would be to continue the research of Sticking Points up into the older grades. If the theory of progression holds, earlier-grade Sticking Points should cause older-grade Sticking Points, thus causing problem areas to “stack up”. Also known as the “Snowball Effect”, this is seen as a single problem area (for example, in grade 1) causing one or more problems in the next grade (e.g., grade 2), which in turn causes one or more problems in the next grade (e.g., grade 3), and so on until the student’s current grade. If this theory of progression holds, a student could theoretically have difficulty in an area of mathematics merely because of a misconception or non-understanding of a concept from a primary grade.

A third area of possible future study would be to compare research of the grade-level Sticking Points and the theory behind the Adaptive-based programming of the new Smarter Balanced test of the Common core. The adaptive nature of the test is meant to
present students with questions based on how they answered previous questions. In this way, if a student enters an incorrect answer to a sixth-grade question, they should be presented with a question which is meant to measure their understanding of the previous standard in the progression of standards. Research into the Smarter Balanced Assessment’s adaptive-based programming and a comparison to further research into grade-level Sticking Points has the potential to verify the program’s progression and/or verify the relevance of the grade-level Sticking Points.

A fourth area of possible future study would be to interview a broader base of teachers in order to get a more well-rounded idea of common Sticking Points. Possible interview bases include teachers being interviewed district-wide, SES-based, across SES areas, etc. Along the same line, further research into the effectiveness of specific teaching strategies for each given Sticking Point would be beneficial while interviewing broader bases of teachers.
Part II
Website Screenshots

The following pages contain screenshots of the website “Common Core Math”, created using the previous research and writings found in this thesis. The website can be found using this link:  
http://commoncoremathparents.weebly.com

Common Core Math

IN THE NEWS

A Common Problem: Examining new standards for Oregon students

What are the Standards?

4th Grade

Grade Four

Sticking Points

5th Grade

Grade Five

Sticking Points

See the Common Core State Standards for Mathematics in Oregon. These are the written statements of what your student's teachers will be expecting them to learn in math this year.
Common Core Math

A Guide for Grade 4 and 5 Parents

The standards are many.

Keep Calm
and
Parent On

- Khan Academy has a Knowledge Map that shows visually how math topics are all interconnected. Check it out here.
- The IXL Learning website has an explanation of how their site aligns with the new Common Core standards. Read it here.
- The Common Core State Standards Initiative website has a page on “What Parents Should Know”. See it here.

The Common Core State Standards Initiative website has a link to a video (also found on YouTube, posted by dcpublicschools). Watch it below.
The Council of the Great City Schools has published Parent Roadmaps to help introduce parents to the changes in school standards. Below are the Roadmaps (posted with permission from CGCS) in both English and Spanish for grades 4 and 5. The Roadmaps for kindergarten through high school can be found here.
guía para
LOS PADRES
APOYANDO A SU HIJO EN QUINTO GRADO
MATEMÁTICAS
Grade 4 Sticking Points

"Sticking Points" are the skills/standards which students consistently have difficulty mastering.

- Visualizing word problems (both single-step and multi-step) — OA.2, OA.3
- Knowing what operation to use in word problems — OA.2
- Verifying and assessing the reasonableness of answers — OA.3
- Finding factor pairs of numbers 1-100 — OA.4
- Rounding multi-digit numbers to any place — NBT.3
- Subtracting multi-digit whole numbers, with borrowing — NBT.4
- Long division of 4-digit numbers by a single-digit number — NBT.6
- Comparing fractions with different denominators — NF.2
- Converting between units — MD.2
Visualizing Word Problems and Knowing What Operation to Use
Grade 4 — 4.OA.2

"Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison."
- CCSS 4.OA.2

Expectations for Students
Students must be able to multiply and divide.
For example: 6 x 7 = 42
7 x 6 = 42
42 / 6 = 7
42 / 7 = 6

Students must be able to interpret word problems to determine which operation will be required.
For example: Key phrases such as "part of a whole" or "part of a part" would indicate multiplication

From the Teachers
In an interview, fourth grade teachers noted that students commonly have difficulty with visualizing word problems (creating pictures of the situation/story being described in the problem), as well as with knowing which operation to use (multiplication vs. division).

Progression
The previous three relevant standards
4.OA.1 - "Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations."

3.OA.6 - "Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8."

3.OA.3 - "Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem."

The next relevant standard
5.OA.2 - "Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product."
Resources

Websites

- Intro to Multiplication* [video] [Arithmetic and Pre-Algebra]
- Multiplication as groups of objects* [video] [Arithmetic and Pre-Algebra]
- More on the concept of multiplication* [video] [Arithmetic and Pre-Algebra]
- The idea of division* [video] [Arithmetic and Pre-Algebra]
- Basic Multiplication* [video] [Arithmetic and Pre-Algebra]
- Multiplying 1 Digit Numbers* [video] [Arithmetic and Pre-Algebra]
- Multiplication 2: The multiplication table* [video] [Arithmetic and Pre-Algebra]
- Example: Ways to represent multiplication* [video] [Arithmetic and Pre-Algebra]
- Multiplication 3: 10, 11, 12 times table* [video] [Arithmetic and Pre-Algebra]
- Division * [video] [Arithmetic and Pre-Algebra]
- Basic division* [practice] [Arithmetic and Pre-Algebra]
- 1-digit division* [practice] [Arithmetic and Pre-Algebra]
- Example: Expressing division in multiple ways* [video] [Arithmetic and Pre-Algebra]
- Comparing with multiplication exercise* [video] [Arithmetic and Pre-Algebra]
- Comparing with multiplication* [practice] [Arithmetic and Pre-Algebra]
- Comparing ages* [video] [Arithmetic and Pre-Algebra]
- Multiplication word problem example 1* [video] [Arithmetic and Pre-Algebra]
- Division word problem example 1* [video] [Arithmetic and Pre-Algebra]
- Comparing magic strength* [video] [Arithmetic and Pre-Algebra]
- Multiplication and division word problems* [practice] [Arithmetic and Pre-Algebra]

See a more complete list of related Khan Academy videos and practice pages here

IXL

- Multiplication Sentences* [practice] [grade 2]
- Multiplication Tables up to 5* [practice] [grade 2]
- Multiplication Tables up to 10* [practice] [grade 2]
- Divisors and Quotients up to 9 [practice] [grade 2]
- Divisors and Quotients up to 10* [practice] [grade 2]

Other grade two practice pages

- Multiply a one-digit number by a larger number - word problems* [practice] [grade 3]
- Multiply three or more numbers - word problems* [practice] [grade 3]
- Division word problems - facts to 10* [practice] [grade 3]
- Divide three-digit numbers - word problems* [practice] [grade 3]
- Divide larger numbers - word problems* [practice] [grade 3]
- Addition, subtraction, multiplication, and division word problems* [practice] [grade 3]
- Multi-step word problems* [practice] [grade 3]

Other grade three practice pages

- Multiply a 2-digit number by a 2-digit number: word problems* [practice] [grade 4]
- Multiply a 2-digit number by a larger number: word problems* [practice] [grade 4]
- Division word problems - facts to 12: word problems* [practice] [grade 4]
- Divide larger numbers, one-digit divisors: word problems* [practice] [grade 4]
- Divide numbers ending in zeroes, multi-digit dividends: word problems* [practice] [grade 4]
- Divide larger numbers by 2-digit numbers: word problems* [practice] [grade 4]
- Addition, subtraction, multiplication, and division word problems* [practice] [grade 4]
- Multi-step word problems* [practice] [grade 4]

Other grade four practice pages

iPad Apps

- Khan Academy - Get the App [all grades]
- IXL - Get the App [all grades]
Common Core Math

Visualizing Word Problems and Verifying and Assessing the Reasonableness of Answers
Grade 4 -- 4.OA.3

"Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding."

- CCSS 4.OA.3

Expectations for Students

Students must be able to solve multistep word problems.
For example: "Seven more than twice ten" requires two operations -- multiplication, then addition
(10 × 2) + 7 = (20) + 7 = 27
For example: "Three less than half the difference of nine and one" requires three operations -- subtraction, then division, then subtraction again
(9 - 1) / 2 - 3 = (8) / 2 - 3 = 4 - 3 = 1

Students must be able to find remainders.
For example: 10 / 6 = 1 remainder 4

Students must be able to interpret remainders.
For example: 10 / 6 = 1 remainder 4
A quotient of 1 remainder 4 means one whole group of six can be made, with four left over that cannot form a whole group of six

Students must be able to use variables to create expressions when solving multistep word problems.
For example: "Three less than half the difference of a number and one"
(n - 1) / 2 - 3
For example: "Twice the sum of seven and a number"
2((x) + 7)

Students must be able to recognize whether their answer is reasonable.
For example: 10 is NOT a reasonable answer for 2 x .5
Multiplying 2 by a number less than one (.5) will result in a number that is smaller than 2.

From the Teachers

In an interview, fourth grade teachers noted that students commonly have difficulty with visualizing word problems (creating pictures of the situation/story being described in the problem), as well as with verifying and assessing the reasonableness of answers (recognizing whether an answer makes sense based on the context of the problem).
Progression

The previous three relevant standards

**4.NBT.6** - “Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.”

**4.NBT.5** - “Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.”

**3.OA.7** - “Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 x 5 = 200 or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.”

The next relevant standard

**5.NBT.5** - “Fluently multiply multi-digit whole numbers using the standard algorithm.”

Resources

**Websites**

Khan Academy
- “Multiplication and division word problems” [practice] [grade 4]
- “Multi-step word problems with whole numbers exercises 1” [video] [grade 4]
- “Multi-step word problems with whole numbers exercises 2” [video] [grade 4]
- “Multi-step word problems with whole numbers exercises 3” [video] [grade 4]
- “Multi-step word problems with whole numbers exercises 4” [practice] [grade 4]
- “How many cars can fit in the parking lot” [video] [grade 3]
- “Bars of soda for the party” [video] [grade 3]
- “Blueberries for friends” [video] [grade 3]
- “Average height of a building’s floor” [video] [grade 3]
- “How many truffle eating guests attended a party?” [video] [grade 3]
- “Total seats in a theater” [video] [grade 3]
- “Marbles for friends” [video] [grade 3]
- “Running distance in a week” [video] [grade 3]
- “Two-step word problems with addition, subtraction, multiplication, and division” [practice] [grade 3]

**iPad Apps**

Khan Academy
- Get the App [all grades]

XL
- Get the App [all grades]
Finding factor pairs of numbers 30 - 100
Grade 4 -- 4.OA.4

"Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite."
- CCSS 4.OA.4

Expectations for Students

Students must be able to find the factor pair(s) for each number from 1 to 100.
For example: 10 has two factor pairs
1 × 10 = 10 This means that 1 and 10 create one factor pair of 10
2 × 5 = 10 This means that 2 and 5 create the second factor pair of 10

Students must be able to recognize that every factor of a number can be multiplied by some number to get the original number.
For example: 2 is a factor of 10
2 can be multiplied by another number (5) to get 10
This means that 10 is a multiple of its factor

Students must be able to determine whether any of the single-digit numbers (1 - 9) are factors of any given number from 1 to 100.
For example: 10 is a number from 1 to 100
3 is a single-digit number
3 is not a factor of 10 because there is no whole number which, when multiplied by 3, gets 10

Students must be able to determine whether a given number from 1 to 100 has one (prime) or more (composite) factor pairs.
For example: 10 is a number from 1 to 100
10 has two factor pairs: 1 × 10, and 2 × 5
Because 10 has more than one factor pair, it is composite

For example: 11 is a number from 1 to 100
11 has only one factor pair: 1 × 11
Because 11 only has one factor pair, it is prime

From the Teachers

In an interview, fourth grade teachers noted that students commonly have difficulty with finding the factor pairs of numbers 30 to 100.
Progression

The previous three relevant standards

4.OA.1 - “Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 * 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.”

3.OA.7 - “Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 * 5 = 40, one knows 40 / 5 = 8 or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.”

3.OA.5 - “Apply properties of operations as strategies to multiply and divide. 2 Examples: If 6 * 4 = 24 is known, then 6 * 6 = 24 is also known. (Commutative property of multiplication.) 3 * 5 * 2 can be found by 3 * 5 = 15, then 15 * 2 = 30, or by 5 * 2 = 10, then 3 * 10 = 30. (Associative property of multiplication.) Knowing that 8 * 5 = 40 and 8 * 2 = 16, one can find 8 * 7 as 8 * (5 + 2) = (8 * 5) + (8 * 2) = 40 + 16 = 56. (Distributive property.)”

The next relevant standard

6.NS.4 - “Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).”

Resources

Websites

Khan Academy
- “Finding factors of a number” [video] [grade 4]
- “Finding factors and multiples” [video] [grade 4]
- “Prime numbers” [video] [grade 4]
- “Recognizing prime numbers” [video] [grade 4]
- “Prime numbers” [practice] [grade 4]
- “Composite numbers” [practice] [grade 4]
- “Divisibility G.5” [practice] [grade 4]

IXL
- “Missing factors - facts to 12” [practice] [grade 4]
- “Choose the multiples of a given number up to 12” [practice] [grade 4]
- “Identify factors” [practice] [grade 4]
- “Missing factors - facts to 12” [practice] [grade 3]
- “Prime and composite numbers” [practice] [grade 4]

iPad Apps

Khan Academy
- Get the App [all grades]

IXL
- Get the App [all grades]
Common Core Math

Rounding multi-digit numbers to any place
Grade 4 -- 4.NBT.3

"Use place value understanding to round multi-digit whole numbers to any place."
- CCSS 4.NBT.3

Expectations for Students

Students must be able to name the place value of any digit.
For example:
- In the number 1,489,723
  - The 3 is in the ones place
  - The 2 is in the tens place
  - The 7 is in the hundreds place
  - The 9 is in the thousands place
  - The 8 is in the ten-thousands place
  - The 4 is in the hundred-thousands place
  - The 1 is in the millions place

Students must be able to round multi-digit whole numbers to any given place.
For example:
- 47,325 rounded to the nearest ten-thousand is 50,000
- 47,325 rounded to the nearest thousand is 47,000
- 47,325 rounded to the nearest hundred is 47,300
- 47,325 rounded to the nearest ten is 47,330

From the Teachers

In an interview, fourth grade teachers noted that students commonly have difficulty with rounding multi-digit numbers to any place.

Progression

The previous two relevant standards:

4.NBT.2 - "Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons."

4.NBT.1 - "Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 = 70 = 7 by applying concepts of place value and division."
The next two relevant standards

4.NF.7 - “Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.”

5.NBT.3b - “Read, write, and compare decimals to thousandths. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.”

Resources

Websites

- Khan Academy
  - "Place value 1 [video] [grade 4]
  - "Place value 2 [video] [grade 4]
  - "Place value 3 [video] [grade 4]
  - "Place value [practice] [grade 4]
  - "Comparing place values [video] [grade 4]
  - "Understanding place value 1 exercise [video] [grade 4]
  - "Understanding place value [practice] [grade 4]
  - "Regrouping numbers into various place values [video] [grade 4]
  - "Comparing whole number place values [video] [grade 4]
  - "Largest possible number from 4 digits example [video] [grade 4]
  - "Understanding whole number representations [practice] [grade 4]
  - "Rounding whole numbers 1 [video] [grade 4]
  - "Rounding whole numbers 2 [video] [grade 4]
  - "Rounding whole numbers 3 [video] [grade 4]
  - "Rounding whole numbers [practice] [grade 4]
  - "Rounding to the nearest 10 [video] [grade 3]
  - "Rounding to the nearest 100 [video] [grade 3]
  - "Examples rounding to the nearest 10 and 100 [video] [grade 3]
  - "Rounding to the nearest ten or hundred [practice] [grade 3]

IXL

- "Place value [practice] [grade 4]
- "Convert between place values [practice] [grade 4]
- "Word names for numbers [practice] [grade 4]
- "Rounding [practice] [grade 4]
- "Write numbers in words [practice] [grade 3]
- "Place value models [practice] [grade 3]
- "Place value names [practice] [grade 3]
- "Value of a digit [practice] [grade 3]
- "Identify the digit with a particular place value [practice] [grade 3]
- "Convert to/from a number [practice] [grade 3]
- "Convert between place values [practice] [grade 3]
- "Convert from expanded form [practice] [grade 3]
- "Convert between standard and expanded form [practice] [grade 3]
- "Place value word problems [practice] [grade 3]

iPad Apps

Khan Academy
- Get the App [all grades]

IXL
- Get the App [all grades]
Common Core Math

Subtracting multi-digit whole numbers, with borrowing
Grade 4 -- 4.NBT.4

"Fluently add and subtract multi-digit whole numbers using the standard algorithm."
- CCSS 4.NBT.4

Expectations for Students

Students must be able to add multi-digit whole numbers using the standard algorithm.
For example:

\[
\begin{align*}
9 & \quad 4 & \quad 5 & \quad 0 & \quad 3 \\
+ & \quad 3 & \quad 4 & \quad 2 & \quad 1 \\
\hline
9 & \quad 7 & \quad 9 & \quad 3 & \quad 4
\end{align*}
\]

Students must be able to carry while adding multi-digit whole numbers using the standard algorithm.
For example:

\[
\begin{align*}
1 & \quad 1 & \quad 1 & \quad 1 & \quad 1 \\
9 & \quad 4 & \quad 5 & \quad 0 & \quad 3 \\
+ & \quad 8 & \quad 7 & \quad 9 & \quad 9 \\
\hline
1 & \quad 0 & \quad 3 & \quad 3 & \quad 0 & \quad 2
\end{align*}
\]

Students must be able to subtract multi-digit whole numbers using the standard algorithm.
For example:

\[
\begin{align*}
9 & \quad 4 & \quad 5 & \quad 7 & \quad 3 \\
- & \quad 3 & \quad 4 & \quad 3 & \quad 1 \\
\hline
9 & \quad 1 & \quad 1 & \quad 4 & \quad 2
\end{align*}
\]

Students must be able to borrow while subtracting multi-digit whole numbers using the standard algorithm.
For example:

\[
\begin{align*}
1 & \quad 2 & \quad 3 & \quad 4 & \quad 5 \\
9 & \quad 4 & \quad 5 & \quad 6 \\
\hline
7 & \quad 6 & \quad 8 & \quad 6 & \quad 9
\end{align*}
\]

From the Teachers

In an interview, fourth grade teachers noted that students commonly have difficulty with borrowing while subtracting multi-digit whole numbers due to a lack of consistent organization when re-writing problems (i.e., students do not keep within straight place value columns).

Progression
The previous two relevant standards

3.NBT.2. “Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.”

2.NBT.5. “Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.”

The next relevant standard

4.NBT.5. “Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.”

Resources

Websites

Khan Academy
- “Base ten warmup” [video] [grade 4]
- “Carrying when adding three-digit numbers” [video] [grade 4]
- “Why carrying works” [video] [grade 4]
- “Examples of regrouping when adding three digit numbers” [video] [grade 4]
- “Addition within 1000” [practice] [grade 4]
- “Basic regrouping when subtracting three digit numbers” [video] [grade 4]
- “Regrouping when subtracting three digit numbers” [video] [grade 4]
- “Regrouping twice when subtracting three digit numbers” [video] [grade 4]
- “Regrouping from 0 when subtracting three digit numbers” [video] [grade 4]
- “Subtraction within 1000” [practice] [grade 4]
- “Mental technique for subtraction without regrouping” [video] [grade 3]

IXL
- “Add numbers up to millions” [practice] [grade 4]
- “Add numbers up to millions: word problems” [practice] [grade 4]
- “Addition: fill in the missing digits” [practice] [grade 4]
- “Properties of addition” [practice] [grade 4]
- “Add 3 or more numbers up to millions” [practice] [grade 4]
- “Addition patterns over increasing place values” [practice] [grade 4]
- “Choose numbers with a particular sum” [practice] [grade 4]
- “Estimate sums” [practice] [grade 4]
- “Estimate sums: word problems” [practice] [grade 4]
- “Subtract numbers up to millions” [practice] [grade 4]
- “Subtract numbers up to millions: word problems” [practice] [grade 4]
- “Subtraction: fill in the missing digits” [practice] [grade 4]
- “Subtraction patterns over increasing place values” [practice] [grade 4]
- “Choose numbers with a particular difference” [practice] [grade 4]
- “Estimate differences” [practice] [grade 4]
- “Estimate differences: word problems” [practice] [grade 4]
- “Complete the addition sentence - up to three digits” [practice] [grade 3]
- “Add three or more numbers up to three digits each” [practice] [grade 3]
- “Add two numbers with four or more digits” [practice] [grade 3]
- “Complete the addition sentence - four or more digits” [practice] [grade 3]
- “Add three or more numbers with four or more digits” [practice] [grade 3]
- “Addition: fill in the missing digits” [practice] [grade 3]
- “Subtract numbers up to three digits” [practice] [grade 3]
- “Complete the subtraction sentence - up to three digits” [practice] [grade 3]
- “Subtract numbers with four or more digits” [practice] [grade 3]
- “Subtraction: fill in the missing digits” [practice] [grade 3]

iPad Apps

Khan Academy - Get the App [all grades]
IXL - Get the App [all grades]
Long division of 4-digit numbers by a single-digit number
Grade 4 -- 4.NBT.6

*Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.*

- CCSS 4.NBT.6

Expectations for Students

Students must be able to divide four-, three-, two-, and one-digit numbers by one-digit numbers.

For example:
- 6381 divided by 3 equals 2127
- 381 divided by 3 equals 127
- 81 divided by 3 equals 27
- 9 divided by 3 equals 3

Students must be able to use strategies based on place value, such as the standard algorithm.

For example:

```
  2 1 2 7
3 | 6 3 8 1
  - 5 4
  ---
  3 8
  - 3
  --
  0 8
  - 6
  --
  2 1
  -2 1
  ---
   0 0
```

Students must be able to use strategies based on the relationship between multiplication and division (i.e., “fact families”).

For example:
- 6381 divided by 3 equals 2127
- 6381 divided by 2127 equals 3
- 3 multiplied by 2127 equals 6381
- 2127 multiplied by 3 equals 6381

Students must be able to use equations, rectangular arrays, and area models to illustrate a division problem.

For example:
- 6381 divided by 3 equals 2127 can be written as 6381 / 3 = 2127

For example:
- Rectangular arrays
- Area models

Students must be able to use equations, rectangular arrays, and area models to illustrate a division problem.

For example:
- 6381 divided by 3 equals 2127

Students must be able to find remainders.

For example:
- 10 / 6 = 1 remainder 4

Students must be able to interpret remainders.

For example:
- 10 / 6 = 1 remainder 4

A quotient of 1 remainder 4 means one whole group of six can be made, with four left over that cannot form a whole group of six.
From the Teachers

In an interview, fourth grade teachers noted that students commonly have difficulty with long division of 4-digit numbers by a single-digit number.

Progression

The previous relevant standard: 4.NBT.6 - "Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models."

The next two relevant standards:

4.OA.3 - "Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding."

5.NBT.5 - "Fluently multiply multi-digit whole numbers using the standard algorithm."

Resources

Websites

Khan Academy
- "Introduction to long division" [video] (arithmetic and pre-algebra)
- "Multi-digit division without remainders" [practice] (arithmetic and pre-algebra)
- "Overview of long division" [video] (arithmetic and pre-algebra)
- "Long division with remainder examples" [video] (arithmetic and pre-algebra)
- "More long division with and without remainders" [video] (arithmetic and pre-algebra)
- "Division with remainders" [practice] (arithmetic and pre-algebra)
- "Dividing by a two digit number" [video] (arithmetic and pre-algebra)
- "Divide by 2 digits" [practice] (arithmetic and pre-algebra)
- "Multi-digit division" [practice] (arithmetic and pre-algebra)
- "Level 1 division" [video] (arithmetic and pre-algebra)
- "Understanding multiplication through area models" [video] (arithmetic and pre-algebra)
- "Area model for multiplication" [video] (arithmetic and pre-algebra)

IXL
- "Properties of division" [practice] (grade 4)
- "Divide by 2-digit numbers by multiples of 10" [practice] (grade 4)
- "Divide by 2-digit numbers" [practice] (grade 4)
- "Divide larger numbers by 2-digit numbers" [practice] (grade 4)
- "Divide three-digit numbers" [practice] (grade 3)
- "Divide three-digit numbers - word problems" [practice] (grade 3)
- "Divide larger numbers" [practice] (grade 3)

Other
- "Area Models for Multiplication and Division" [Interactive model; requires Flash] (Number and Operations)

iPad Apps

Khan Academy - Get the App [all grades]
IXL - Get the App [all grades]
Comparing fractions with different denominators

Grade 4 -- 4.NF.2

"Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model."

- CCSS 4.NF.2

Expectations for Students

Students must be able to find equivalent fractions.
For example: 1/2 = 2/4 = 3/6 = 4/8 = 5/10 = 6/12 = 7/14 = 8/16 = 9/18 = 10/20
For example: 1/3 = 2/6 = 3/9 = 4/12 = 5/15 = 6/18 = 7/21 = 8/24 = 9/27 = 10/30
For example: 1/4 = 2/8 = 3/12 = 4/16 = 5/20 = 6/24 = 7/28 = 8/32 = 9/36 = 10/40

Students must be able to compare fractions with like denominators.
For example: 1/4 is less than 3/4 2/4 is equal to 1/2 3/4 is greater than 1/4

Students must be able to compare fractions with like denominators.
For example: 0, 1/4, 1/3, 1/2, 2/3, 3/4, 1 are benchmark fractions

Students must be able to recognize benchmark fractions with ease.
For example: When comparing 4/7 and 5/11
4/7 is more than 1/2
5/11 is less than 1/2
4/7 must be greater than 5/11

Students must be able to recognize that two fractions must be part of the same whole in order to compare them with each other.
For example: Just as 1/2 of a chair is equal to 1/2 of a couch, 1/2 of 4 is not equal to 1/2 of 5
1/2 of 4 = 2 1/2 of 5 = 3

Students must be able to use symbols >, =, and < to compare fractions.
For example: 1/4 < 3/4 2/4 = 2/4 3/4 > 1/4

Students must be able to justify their conclusions by using visual models.
For example: Fraction models **Interactive**
Models for the Multiplication and Division of Fractions **Interactive; requires Flash**

From the Teachers

In an interview, fourth grade teachers noted that students commonly have difficulty with comparing fractions with different denominators, due to a lack of persistence and perseverance.
Progression

3.NF.3a - "Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model."

3.NF.3b - "Understand a fraction as a number on the number line; represent fractions on a number line diagram. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line."

5.NF.1 - "Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (in general, a/b + c/d = (ad + bc)/bd)"

Resources

Websites
- Khan Academy - "Introduction to equivalent fractions" [video] [grade 4]
- "Visualizing equivalent fractions" [video] [grade 4]
- "Equivalent fraction word problem example" [video] [grade 4]
- "Equivalent fraction word problem example 2" [video] [grade 4]
- "Equivalent fraction word problem example 3" [video] [grade 4]
- "Visualizing equivalent fractions" [practice] [grade 4]
- "More on equivalent fractions" [video] [grade 4]
- "Equivalent fractions example" [video] [grade 4]
- "Equivalent fractions" [practice] [grade 4]
- "Fractions cut and copy 1 exercise" [practice] [grade 4]
- "Fractions cut and copy 1" [practice] [grade 4]
- "Comparing fractions 2" [video] [grade 4]
- "Comparing fractions 3" [practice] [grade 4]
- "Ordering fractions" [video] [grade 4]
- "Ordering fractions" [practice] [grade 4]

IXL
- "Equivalent fractions" [practice] [grade 4]
- "Fractions with denominators of 10, 100, and 1000" [practice] [grade 4]
- "Patterns of equivalent fractions" [practice] [grade 4]
- "Reduce fractions to lowest terms" [practice] [grade 4]
- "Fractions review" [practice] [grade 4]
- "Compare fractions" [practice] [grade 4]
- "Put fractions in order" [practice] [grade 4]
- "Unit fraction review" [practice] [grade 3]
- "Fraction review" [practice] [grade 3]
- "Fraction review - word problems" [practice] [grade 3]
- "Fractions on number line" [practice] [grade 3]
- "Compare fractions" [practice] [grade 3]
- "Equivalent fractions: type the missing numerator or denominator" [practice] [grade 3]
- "Equivalent fractions: choose the equivalent fraction" [practice] [grade 3]
- "Fractions with denominators of 10 and 100" [practice] [grade 3]
- "Order fractions with like numerators" [practice] [grade 3]
- "Order fractions with like denominators" [practice] [grade 3]
- "Order fractions" [practice] [grade 3]

iPad Apps
- Khan Academy - Get the App [all grades]
- IXL - Get the App [all grades]
Common Core Math

Converting Between Units
Grade 4 -- 4.MD.2

"Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale."

- CCSS 4.MD.2

Expectations for Students

Students must be able to add, subtract, multiply, and divide distances.
For example:
- 2 feet + 2 feet = 4 feet
- 5 inches + 3 inches = 2 inches
- 15 meters x 2 = 30 meters
- 15 meters x 2 meters = 30 square meters
- 20 miles / 2 miles = 10
- 20 miles / 2 = 10 miles
- 20 square miles / 2 miles = 10 miles

Students must be able to add, subtract, multiply, and divide intervals of time.
For example:
- 25 seconds + 13 seconds = 38 seconds
- 13 minutes - 5 minutes = 8 minutes
- 6 minutes x 4 = 24 minutes
- 60 minutes / 10 = 6 minutes
- 60 minutes / 10 minutes = 6

Students must be able to add, subtract, multiply, and divide liquid volumes.
For example:
- 2 gallons + 5 gallons = 7 gallons
- 10 litres - 4 litres = 6 litres
- 3 pints x 4 = 12 pints
- 15 quarts / 3 = 5 quarts
- 15 quarts / 3 quarts = 5

Students must be able to add, subtract, multiply, and divide masses of objects.
For example:
- 2kg + 7 kg = 9 kg
- 25lbs - 15lbs = 10 lbs
- 14g x 3 = 42 g
- 16 ounces / 4 = 4 ounces
- 16 ounces / 4 ounces = 4

Students must be able to add, subtract, multiply, and divide money.
For example:
- 50 cents + 25 cents = 75 cents
- $20 - 52 = $18
- 4 dimes X 10 = 40 dimes
- 24 quarters / 4 = 6 quarters
- 24 quarters / 4 quarters = 6

Students must be able to add, subtract, multiply, and divide fractions.
For example:
- 3/4 + 3/4 = 6/4
- 5/7 - 2/7 = 3/7
- 1/4 x 2/3 = 2/12
- 4/7 / 3/4 = 4/7 x 4/3 = 16/21
Students must be able to add, subtract, multiply, and divide decimals.

For example:

- $1.4 + 2.7 = 4.1$
- $5.8 - 3.5 = 2.3$
- $1.2 \times 3.0 = 3.6$
- $4.8 \div 1.2 = 4.0$

Students must be able to convert between units within the same measurement “family”.

For example:

- 1 foot = 12 inches, 1 meter = 100 centimeters
- 1 minute = 60 seconds, 1 hour = 60 minutes, 1 day = 24 hours, 1 week = 7 days
- 1 quart = 2 pints, 1 gallon = 4 quarts, 1 cup = 16 oz
- 1 lb = 16 ounces, 1 kilogram = 1,000 grams
- $1 = 100$ cents = 100 pennies = 20 nickels = 10 dimes = 4 quarters

Students must be able to use number lines to model measurements.

For example: Number Line **Interactive**

Postage Rates **Interactive Game**

---

**From the Teachers**

In an interview, fourth grade teachers noted that students commonly have difficulty with converting between units (for example: converting minutes to hours, inches to feet, etc.).

---

**Progression**

**The previous three relevant standards**

4.MD.1 - “Know relative sizes of measurement units within one system of units including km, m, cm; kg, g, lb; oz, l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit; record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...”

3.MD.2 - “Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.”

**The next relevant standard**

5.MD.1 - “Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems.”

---

**Resources**
Websites

Khan Academy
- "Converting between units of time" [video] [grade 4]
- "Converting feet to inches" [video] [grade 4]
- "Converting distances in the metric system" [video] [grade 4]
- "Converting fluid volume in US customary units" [video] [grade 4]
- "Converting weight in metric and US customary systems" [video] [grade 4]
- "Measurement units" [practice] [grade 4]
- "Time to leave for home" [video] [grade 4]
- "Time before volleyball practice" [video] [grade 4]
- "Measuring time word problems" [practice] [grade 4]
- "Change from buying apples" [video] [grade 4]
- "Currency conversion" [video] [grade 4]
- "Measuring and converting money word problems" [practice] [grade 4]
- "Left over tea from party" [video] [grade 4]
- "Jogging and walking" [video] [grade 4]
- "Measurement word problems with metric units" [practice] [grade 4]
- "Running laps" [video] [grade 4]
- "Safe elevator" [video] [grade 4]
- "Blood drive" [video] [grade 4]
- "Measurement word problems with US customary units" [practice] [grade 4]

IXL
- "Add and subtract money amounts" [practice] [grade 4]
- "Add, subtract, multiply, and divide money amounts" [practice] [grade 4]
- "Making change" [practice] [grade 4]
- "Convert mixed customary units" [practice] [grade 4]
- "Add and subtract mixed customary units" [practice] [grade 4]
- "Convert time units" [practice] [grade 4]
- "Add and subtract mixed time units" [practice] [grade 4]
- "Add and subtract money amounts" [practice] [grade 3]
- "Multiply money amounts" [practice] [grade 3]
- "Divide money amounts" [practice] [grade 3]
- "Relate time units" [practice] [grade 3]
- "Convert between hours and fractions of hours" [practice] [grade 3]
- "Compare and convert customary units of weight" [practice] [grade 3]
- "Compare and convert customary units of volume" [practice] [grade 3]
- "Conversion tables - customary units" [practice] [grade 3]
- "Compare and convert metric units of weight" [practice] [grade 3]
- "Compare and convert metric units of volume" [practice] [grade 3]
- "Conversion tables - metric units" [practice] [grade 3]

iPad Apps

Khan Academy - Get the App [all grades]
RL - Get the App [all grades]
Common Core Math

Grade 5 Sticking Points

"Sticking Points" are the skills/standards which students **consistently** have difficulty mastering.

- Order of Operations — OA.1
- Multiplying powers of 10 — NBT.2
- Comparing decimals — NBT.3a, NBT.3b
- Rounding, to both the left and right of the decimal — NBT.4
- Long division, dividing by two-digit numbers — NBT.6
- Adding and subtracting fractions with unlike denominators — NF.1
- Knowing what operation to use in word problems — NF.2
- Converting between units — MD.1
- Finding volume of irregular figures — MD.3b, MD.4
- Graphing on the four-quadrant coordinate plane — G.2
- Naming shapes/figures based on attributes — G.3
Common Core Math

Order of Operations
Grade 5 — 5.OA.1

"Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols."

- CCSS 5.OA.1

Expectations for Students

Students must be able to use the Order of Operations to evaluate expressions.

For example: P - Parentheses
- E - Exponents
- M/D - Multiplication and Division from left to right
- A/S - Addition and Subtraction from left to right

Students must be able to use parentheses (brackets/braces) in expressions.

For example: Five less the sum of two and one = 5 - (2 + 1)

Students must be able to evaluate expressions which use parentheses (brackets/braces)

For example:
- 5 - (2 + 1) =
- 5 - 3 =
- 2

For example:
- 2 (5 - 3) + (3 x 4) =
- 2 (2) + (12) =
- 4 + 12 =
- 16

From the Teachers

In an interview, fifth grade teachers noted that students commonly have difficulty with following/remembering the Order of Operations (PEMDAS).

Progression

The previous relevant standard

3.OA.8 - "Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding."

The next two relevant standards

5.OA.2 - "Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product."
6.EE.2a: "Write, read, and evaluate expressions in which letters stand for numbers. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation ‘Subtract y from 5’ as 5 – y."

Resources

**Websites**

- Khan Academy
  - "Introduction to order of operations" [video] [grade 6]
  - "Order of operations" [video] [grade 6]
  - "More complicated order of operations example" [video] [grade 6]
  - "Order of operations" [practice] [grade 6]
  - "Evaluating expressions with exponents" [practice] [grade 6]
  - "Evaluating an expression with and without parentheses" [video] [grade 5]
  - "Expressions with parentheses" [practice] [grade 5]

**IXL**

- "Simplify expressions using order of operations and parentheses" [practice] [grade 4]
- "Simplify expressions using order of operations and parentheses" [practice] [grade 5]

**iPad Apps**

- Khan Academy - Get the App [all grades]
- IXL - Get the App [all grades]
Multiplying powers of 10
Grade 5 -- 5.NBT.2

*Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.*
- CCSS 5.NBT.2

Expectations for Students

Students must be able to multiply a number by powers of 10
For example: 
3 9 6 7
\[
\begin{array}{c}
3 9 6 7 \\
\times 1 0 0 \\
\hline
3 9 6 7 0 \\
+ 3 9 6 7 0 \\
\hline
3 9 6 7 9 0
\end{array}
\]

Students must be able to explain the patterns in the number of zeros of the product when multiplying by powers of 10.
For example: When multiplying by 100 (also written as 10^2), the original number is shifted two place-value places to the left. 3967 X 100 = 3967000
Zeros fill in the now-empty place-value places.

Students must be able to explain the patterns in the placement of the decimal point when a decimal is multiplied by a power of 10.
For example: When multiplying by 100 (also written as 10^2), the original decimal point is shifted two place-value places to the right.
3967.52 X 100 = 396752
9374.56487 X 100 = 937456487
Any empty place-value places are filled in by zeros.
3967 = 3967.
3967. X 100 = \_ 396700.

Students must be able to explain the patterns in the placement of the decimal point when a decimal is dividing by a power of 10.
For example: When dividing by 100 (also written as 10^2), the original decimal point is shifted two place-value places to the left.
67.2 / 100 = .672
9374.56 / 100 = 93.7456
Any empty place-value places are filled in by zeros.

Students must be able to use whole number exponents to write powers of 10.
For example:
\[10 = 10^1\]
\[100 = 10^2\]
\[1,000 = 10^3\]
\[10,000 = 10^4\]
\[100,000 = 10^5\]
\[1,000,000 = 10^6\]
From the Teachers

In an interview, fifth grade teachers noted that students commonly have difficulty with multiplying powers of 10.

Progression

The previous three relevant standards

5.NBT.1 - “Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.”

5.NBT.2 - “Read, write, and compare decimals to thousandths. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.”

4.NF.7 - “Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.”

The next relevant standard

5.NBT.4 - “Use place value understanding to round decimals to any place.”

6.EE.1 - “Write and evaluate numerical expressions involving whole-number exponents.”

Resources

Websites

Khan Academy - *Comparing place values in decimals* [video] [grade 5]
- *Comparing decimal place value* [practice] [grade 5]
- *Using money to understand decimal place value* [video] [grade 5]
- *“Money and decimal place value intuition”* [practice] [grade 5]
- *“Powers of 10”* [video] [grade 5]
- *“Patterns in zero exercises”* [video] [grade 5]
- *“Patterns in zero practice”* [grade 5]
- *“Multiplying a decimal by a power of 10”* [video] [grade 5]
- *“Introduction to exponents”* [video] [grade 6]
- *“Positive and zero exponents”* [practice] [grade 6]

IXL

- *“Multiply by 10”* [practice] [grade 3]
- *“Divide 2-digit numbers by multiples of 10”* [practice] [grade 4]
- *“Write multiplication expressions using an exponent”* [practice] [grade 6]

iPad Apps

Khan Academy - Get the App [all grades]
IXL - Get the App [all grades]
Comparing Decimals
Grade 5 -- 5.NBT.3a

"Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000)."

-CCSS 5.NBT.3a

Expectations for Students
Students must be able to read and write decimals to the thousandths using base ten numerals.
For example: Spoken - "Twenty-seven and four-hundred sixteen thousandths"
            Base ten numerals - 27.416

Students must be able to read and write decimals to the thousandths using number names.
For example: Base ten numerals - 336,501
            Number names - "three-hundred and thirty-six and five hundred one thousandths"

Students must be able to read and write decimals to the thousandths using expanded form.
For example: Base ten numerals - 235.847
            Expanded form - 2 × 10 + 3 × 10 + 5 × 1 + 8 × (1/10) + 4 × (1/100) + 7 × (1/1000)
            The 2 is in the tens place - 2 × 10
            The 3 is in the ones place - 3 × 1
            The 8 is in the tenths place 8 × (1/10)
            The 4 is in the hundredths place - 4 × (1/100)
            The 7 is in the thousandths place - 7 × (1/1000)

From the Teachers
In an interview, fifth grade teachers noted that students commonly have difficulty with comparing decimals. Students either tend to not be able to read decimals to the thousandths, or they tend to get "tunnel vision" and look only at the digits to the right of the decimal rather than looking at the entire number.

Progression
The previous three relevant standards: 5.NBT.3a - "Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left."
4.NBT.3 - "Use place value understanding to round multi-digit whole numbers to any place."
The next relevant standard

5.NBT.4 - "Use place value understanding to round decimals to any place."

6.NS.3 - "Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation."

Resources

Websites
Khan Academy
- "Place value 1" [video] [Arithmetic and pre-algebra]
- "Place value 2" [video] [Arithmetic and pre-algebra]
- "Place value 3" [video] [Arithmetic and pre-algebra]
- "Place value" [practice] [Arithmetic and pre-algebra]
- "Representing numbers" [video] [Arithmetic and pre-algebra]
- "Comparing place values" [video] [Arithmetic and pre-algebra]
- "Understanding place value 1 exercise" [practice] [Arithmetic and pre-algebra]
- "Understanding place value" [practice] [Arithmetic and pre-algebra]
- "Place value relationships example" [video] [Arithmetic and pre-algebra]

DLE
- "Place values in whole numbers" [practice] [grade 6]
- "Word names for numbers" [practice] [grade 6]
- "Place values" [practice] [grade 5]
- "Convert between place values" [practice] [grade 5]
- "Word names for numbers" [practice] [grade 5]
- "Place values" [practice] [grade 4]
- "Convert between place values" [practice] [grade 4]
- "Word names for numbers" [practice] [grade 4]
- "Write numbers in words" [practice] [grade 3]
- "Convert from expanded form" [practice] [grade 3]
- "Convert from expanded form - up to hundred" [practice] [grade 2]
- "Convert from expanded form - up to thousands" [practice] [grade 2]

iPad Apps
Khan Academy
- Get the App [all grades]

ICL
- Get the App [all grades]
Comparing Decimals
Grade 5 -- 5.NBT.3b

"Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons."
- CCSS 5.NBT.3b

Expectations for Students

Students must be able to read and write decimals to the thousandths using base ten numerals.
For example: Spoken - "Twenty-seven and four-hundred sixteen thousandths"
Base ten numerals - 27.416

Students must be able to read and write decimals to the thousandths using number names.
For example: Base ten numerals - 336.901
Number names - "Three hundred and thirty-six and nine-hundred and one thousandths"

Students must be able to read and write decimals to the thousandths using expanded form.
For example: Base ten numerals - 25.847
Expanded form - 2 X 10 + 5 X 1 + 8 X (1/10) + 4 X (1/100) + 7 X (1/1000)
- The 2 is in the tens place - 2 X 10
- The 5 is in the ones place - 5 X 1
- The 8 is in the tenths place - 8 X (1/10)
- The 4 is in the hundredths place - 4 X (1/100)
- The 7 is in the thousandths place - 7 X (1/1000)

Students must be able to compare decimals to the thousandths place based on the values of the digits in each place value.
For example: Comparing 14.927 and 7.463
- The decimal 14.927 has digits other than zero starting in the tens place
- The decimal 7.463 has digits other than zero starting in the ones place
- The value of any digit in the tens place is greater than the value of any digit in the ones place
- 1 ten is greater than 7 ones
So 14.927 is greater than 7.463

For example: Comparing 3.567 and 4.175
- Both decimals have digits other than zero starting in the ones place
- The digit in the ones place of 3.567 has a value of 3
- The digit in the ones place of 4.175 has a value of 4
- 3 is less than 4, so 3.567 is less than 4.175

For example: Comparing 5.375 and 5.394
- Both decimals have digits other than zero starting in the ones place
- The digit in the ones place of 5.375 has a value of 5
- The digit in the ones place of 5.394 has a value of 5
- 5 is equal to 5, so we must look at the next place-value place to the right
- The digit in the tenths place of 5.375 has a value of .3
- The digit in the tenths place of 5.394 has a value of .3
- .3 is equal to .3, so we must look at the next place-value place to the right
- The digit in the hundredths place of 5.375 has a value of .7
- The digit in the hundredths place of 5.394 has a value of .9
- .7 is less than .9, so 5.375 is less than 5.394
Students must be able to use >, =, and < to compare decimals to the thousandths place.
For example:
14.927 is greater than 7.463
14.927 > 7.463 and 7.468 < 14.927
For example:
3.567 is less than 4.175
2.567 < 4.175 and 4.175 > 3.567
For example:
4.099 is equal to 4.099
4.099 = 4.099

From the Teachers
In an interview, fifth grade teachers noted that students commonly have difficulty with comparing decimals. Students either tend to not be able to read decimals to the thousandths, or they tend to get "tunnel vision" and look only at the digits to the right of the decimal rather than looking at the entire number.

Progression
The previous two relevant standards
4.NF.7 - "Compare two decimals to thousandths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model."

4.NF.6 - "Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100: describe a length as 0.62 meters; locate 0.62 on a number line diagram."

The next two relevant standards
5.NBT.2 - "Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10."

5.NBT.4 - "Use the place value understanding to round decimals to any place."

Resources
Websites
Khan Academy - "Comparing place values in decimals" [video] [grade 5]
- "Comparing decimal place value" [practice] [grade 5]
- "Comparing decimals example 2" [video] [grade 5]
- "Comparing decimals example 4" [video] [grade 5]
- "Comparing decimals 2" [practice] [grade 5]
- "Ordering decimals example 3" [video] [grade 5]
- "Ordering decimals example 4" [video] [grade 5]
- "Ordering decimals" [practice] [grade 5]

IXL
- "What decimal number is illustrated?" [practice] [grade 5]
- "Understanding decimals expressed in words" [practice] [grade 5]
- "Place values in decimal numbers" [practice] [grade 5]
- "Equivalent decimals" [practice] [grade 5]
- "Decimal number lines" [practice] [grade 5]
- "Compare decimal numbers" [practice] [grade 5]
- "Put decimal numbers in order" [practice] [grade 5]
- "Compare decimal numbers" [practice] [grade 4]
- "Put decimal numbers in order" [practice] [grade 4]
- "Number sequences involving decimals" [practice] [grade 4]

iPad Apps
Khan Academy - Get the App [all grades]
IXL - Get the App [all grades]
Rounding, to both the Left and Right of the Decimal
Grade 5 -- 5.NBT.4

"Use place value understanding to round decimals to any place."
- CCSS 5.NBT.4

Expectations for Students

Students must be able to recognize place value names.
For example: For the number 7,509,386,214
- The 7 is in the millions place
- The 5 is in the hundred-thousands place
- The 0 is in the ten-thousands place
- The 9 is in the thousands place
- The 3 is in the hundreds place
- The 8 is in the tens place
- The 6 is in the ones place
- The 2 is in the tenths place
- The 1 is in the hundredths place
- The 4 is in the thousandths place

Students must be able to round numbers to any place to the left of the decimal.
For example: For the number 3,754,352

Rounded to the nearest ten
- The digit in the tens place is a 5: 3,754,352
- If we round down, we would round down to 3,754,000
- If we round up, we would round up to 3,755,000
- The rule: "four or less, let it rest (round down); five or more, raise the score (round up)"
- To follow the rule, look to the place to the right of the tens place
- The digit to the right of the tens place is a 4, so we would round down to 3,750

Rounded to the nearest hundred
- The digit in the hundreds place is a 7: 3,754,352
- If we round down, we would round down to 3,750,000
- If we round up, we would round up to 3,800,000
- The rule: "four or less, let it rest (round down); five or more, raise the score (round up)"
- To follow the rule, look to the place to the right of the hundreds place
- The digit to the right of the hundreds place is a 4, so we would round down to 3,800

Rounded to the nearest thousand
- The digit in the thousands place is a 3: 3,754,352
- If we round down, we would round down to 3,700,000
- If we round up, we would round up to 4,000,000
- The rule: "four or less, let it rest (round down); five or more, raise the score (round up)"
- To follow the rule, look to the place to the right of the thousands place
- The digit to the right of the thousands place is a 7, so we would round up to 4,000
Students must be able to round numbers to any place to the right of the decimal.

For example: For the number 3.754,392

Rounded to the nearest tenths
- The digit in the tenths place is a 3: 3,754.392
- If we round down, we would round down to 3,754.300
- If we round up, we would round up to 3,754.400

The rule: "four or less, let it rest (round down); five or more, raise the score (round up)"

To follow the rule, look to the place to the right of the tenths place
- The digit to the right of the tenths place is a 3, so we would round up to 3,754.4

Rounded to the nearest hundredths
- The digit in the hundredths place is a 5: 3,754.392
- If we round down, we would round down to 3,754.390
- If we round up, we would round up to 3,754.400

The rule: "four or less, let it rest (round down); five or more, raise the score (round up)"

To follow the rule, look to the place to the right of the hundredths place
- The digit to the right of the hundredths place is a 2, so we would round down to 3,754.39

From the Teachers

In an interview, fifth grade teachers noted that students commonly have difficulty with rounding numbers, both to the left and to the right of the decimal.

Progression

5.NBT.3a - "Read, write, and compare decimals to thousandths. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000)."

4.NBT.3 - "Use place value understanding to round multi-digit whole numbers to any place."

The next two relevant standards

5.NBT.7 - "Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used."

6.NS.3 - "Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation."

Resources

Websites
- Khan Academy
  - "Rounding whole numbers 1" [video] [grade 4]
  - "Rounding whole numbers 2" [video] [grade 4]
  - "Rounding whole numbers 3" [video] [grade 4]
  - "Rounding whole numbers" [practice] [grade 4]
  - "Rounding decimals" [video] [grade 5]
  - "Rounding numbers" [practice] [grade 5]

IXL
- "Round decimals" [practice] [grade 6]
- "Round decimals" [practice] [grade 5]
- "Round decimals" [practice] [grade 4]

iPad Apps
- Khan Academy
  - Get the App [all grades]
- IXL
  - Get the App [all grades]
Long Division, Dividing by Two-Digit Numbers
Grade 5 -- 5.NBT.6

"Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models."
- CCSS 5.NBT.6

Expectations for Students

Students must be able to divide four-digit numbers by two-digit numbers using place value strategies such as the standard algorithm.

For example:

\[
\begin{array}{c}
\begin{array}{c}
27 \longdiv{8424} \\
27 \longdiv{8424} \\
\end{array} \\
\begin{array}{c}
632 \\
632 \\
\end{array} \\
\begin{array}{c}
07 \\
054 \\
\end{array} \\
54 \\
54 \\
6 \\
6
\end{array}
\]

Students must be able to divide four-digit numbers by two-digit numbers using the relationship between multiplication and division (i.e., "fact families").

For example:

- 8424 divided by 27 equals 312
- 8424 divided by 312 equals 27
- 27 multiplied by 312 equals 8424
- 312 multiplied by 27 equals 8424

Students must be able to use equations, rectangular arrays, and area models to illustrate a division problem.

For example:

- "8424 divided by 27 equals 312" can be written as 8424 \div 27 = 312
- For example: Rectangular arrays
- For example: Area models

From the Teachers

In an interview, fifth grade teachers noted that students commonly have difficulty with dividing numbers by two-digit numbers when using long division (i.e., the standard algorithm).
Progression

5.NBT.6 - "Fluently multiply multi-digit whole numbers using the standard algorithm."

4.NBT.6 - "Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models."

4.NBT.5 - "Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models."

6.NS.2 - "Fluently divide multi-digit numbers using the standard algorithm."

Resources

Websites

Khan Academy
- "Dividing by two digits with no remainder" [video] [grade 5]
- "Dividing by a two digit number" [video] [grade 5]
- "Division by 2 digits" [practice] [grade 5]
- "Intro to long division without remainders" [video] [grade 4]
- "Example of long division without remainder" [video] [grade 4]
- "Multi-digit division without remainders" [practice] [grade 4]
- "Introduction to remainders" [video] [grade 4]
- "Long division with remainder example" [video] [grade 4]
- "More long division without and with remainders" [video] [grade 4]
- "Division with remainders" [practice] [grade 4]

IXL
- "Divide by 2-digit numbers" [practice] [grade 5]
- "Properties of division" [practice] [grade 4]
- "Properties of multiplication" [practice] [grade 5]
- "Complete the division sentence - facts to 10" [practice] [grade 3]
- "Fact families" [practice] [grade 2]

iPad Apps

Khan Academy
- Get the App [all grades]

IXL
- Get the App [all grades]
Common Core Math

Adding and Subtracting Fractions with Unlike Denominators

Grade 5 -- 5.NF.1

"Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators."
- CCSS 5.NF.1

Expectations for Students

Students must be able to find equivalent fractions.
For example:
- \( \frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16} = \frac{9}{18} = \frac{10}{20} \)
- \( \frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18} = \frac{7}{21} = \frac{8}{24} = \frac{9}{27} = \frac{10}{30} \)
- \( \frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{6}{24} = \frac{7}{28} = \frac{8}{32} = \frac{9}{36} = \frac{10}{40} \)

Students must be able to find a common denominator for any two fractions.
For example:
- \( \frac{4}{7} \) and \( \frac{5}{3} \) can be converted to have a common denominator of 21, because 7 and 3 (the original denominators) multiply to be 21
  - \( \frac{4}{7} \) and \( \frac{5}{3} \) = \( \frac{12}{21} \) and \( \frac{15}{21} \) Also shown as: \( \frac{4\times7}{3\times7} = \frac{28}{21} \) and \( \frac{5\times3}{7\times3} = \frac{15}{21} \)
- \( \frac{7}{3} = \frac{21}{9} \) and \( \frac{5}{7} = \frac{15}{21} \)

Students must be able to convert a mixed number into an improper fraction.
For example:
- \( 1 \frac{2}{3} = 5\frac{1}{3} \) where the mixed number is voiced as "one and two-thirds"
- \( 1 \frac{1}{2} = 2\frac{1}{2} \) where the mixed number is voiced as "one-fourteen and one-half"
  To convert: multiply the whole number by the denominator, then add the numerator. Place that sum over the original denominator.
- \( 1 \frac{1}{2} = (1\times2) + \frac{1}{2} = 2 + \frac{1}{2} = \frac{5}{2} \) where the whole number is 14, the numerator is 1, and the denominator is 2

Students must be able to add fractions with like denominators.
For example:
- \( \frac{3}{7} + \frac{6}{7} = \frac{9}{7} \)
- \( \frac{2}{3} + \frac{5}{2} = \frac{12}{21} + \frac{15}{21} = \frac{27}{21} = \frac{9}{7} \)
- \( \frac{5}{7} + \frac{9}{2} = \frac{5\times2}{7\times2} + \frac{9\times7}{2\times7} = \frac{10}{14} + \frac{87}{14} = \frac{97}{14} \)
- \( \frac{1}{2} \) and \( \frac{1}{3} \) = \( \frac{1+3}{6} = \frac{4}{6} = \frac{2}{3} \)
Students must be able to subtract fractions with like denominators.

For example:

\[
\frac{5}{7} - \frac{4}{7} = \frac{1}{7}
\]

\[
\frac{35}{21} - \frac{12}{21} = \frac{23}{21}
\]

For example:

\[
\frac{14}{1/2} - \frac{1}{2} = \frac{29}{2} - \frac{5}{3} = \frac{87}{6} - \frac{10}{6} = \frac{77}{6}
\]

---

From the Teachers

In an interview, fifth grade teachers noted that students commonly have difficulty with adding and subtracting fractions with unlike denominators.

---

Progression

The previous three relevant standards

4.NF.3a - “Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.”

4.NF.3b - “Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.”

3.NF.3c - “Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.”

The next relevant standard

5.NF.2 - “Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. For example, by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.”
Resources

Websites
- *Adding fractions with unlike denominators* [video] [grade 5]
- *Adding fractions (ex 1)* [video] [grade 5]
- *Adding fractions with unlike denominators* [practice] [grade 5]
- *Subtracting fractions with unlike denominators* [video] [grade 5]
- *Subtracting fractions with unlike denominators* [practice] [grade 5]
- *Introduction to equivalent fractions* [video] [grade 4]
- *Visualizing equivalent fractions* [video] [grade 4]
- *Equivalent fraction word problem example* [video] [grade 4]
- *Equivalent fraction word problem example 2* [video] [grade 4]
- *Equivalent fraction word problem example 3* [video] [grade 4]
- *Visualizing equivalent fractions* [practice] [grade 4]
- *More on equivalent fractions* [video] [grade 4]
- *Equivalent fractions example* [video] [grade 4]
- *Equivalent fractions* [practice] [grade 4]
- *Fractions cut and copy 1 exercise* [practice] [grade 4]
- *Fractions cut and copy 1 exercise* [practice] [grade 4]
- *Fractions review* [practice] [grade 5]
- *Equivalent fractions* [practice] [grade 5]
- *Reduce fractions to lowest terms* [practice] [grade 5]
- *Convert between improper fractions and mixed numbers* [practice] [grade 5]
- *Least common denominator* [practice] [grade 5]
- *Add and subtract fractions with like denominators* [practice] [grade 5]
- *Add and subtract mixed numbers with like denominators* [practice] [grade 5]
- *Add fractions with unlike denominators* [practice] [grade 5]
- *Subtract fractions with unlike denominators* [practice] [grade 5]
- *Add 3 or more fractions with unlike denominators* [practice] [grade 5]
- *Add mixed numbers with unlike denominators* [practice] [grade 5]
- *Subtract mixed numbers with unlike denominators* [practice] [grade 5]
- *Add and subtract fractions with like denominators* [practice] [grade 6]
- *Add and subtract mixed numbers* [practice] [grade 6]
- *Add and subtract fractions* [practice] [grade 7]
- *Add and subtract mixed numbers* [practice] [grade 7]

iPad Apps
- Get the App [all grades]

Khan Academy
- Get the App [all grades]
Knowing what Operation to use in Word Problems
Grade 5 -- 5.NF.2

*Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.*
- CCSS 5.NF.2

Expectations for Students

Students must be able to recognize that two fractions must be part of the same whole in order to compare them with each other.
For example: 1/2 of a chair is not equal to 1/2 of a couch, 1/2 of 4 is not equal to 1/2 of 6
1/2 of 4 = 2 1/2 of 6 = 3

Students must be able to recognize benchmark fractions with ease.
For example: 0, 1/4, 1/2, 3/4, 1 are benchmark fractions

Students must be able to compare fractions to benchmark fractions.
For example: When comparing 4/7 and 5/11
4/7 is more than 1/2
5/11 is less than 1/2
4/7 must be greater than 5/11

Students must be able to add fractions with like denominators.
For example: 3/7 + 6/7 = 9/7
For example: 4/7 + 5/8 =
12/21 + 35/21 =
47/21
For example: (1 2/3) + (14 1/2) =
5/3 + 29/2 =
(5/3)(2/2) + (29/2)(3/3) =
10/6 + 87/6 =
97/6

Students must be able to subtract fractions with like denominators.
For example: 6/9 - 4/9 = 2/9
For example: 5/3 - 4/7 =
35/21 - 12/21 =
23/21
For example: (14 1/2) - (1 2/3) =
29/2 - 5/3 =
(29/2)(3/3) - (5/3)(2/2) =
87/6 - 10/6 =
77/6

Students must be able to interpret word problems to determine which operation will be required.
For example: Key phrases such as "and" or "in addition to" or "altogether" would indicate addition
From the Teachers

In an interview, fifth grade teachers noted that students commonly have difficulty with knowing what operation to use when solving word problems.

Progression

The previous three relevant standards

- 5.NF.1 - “Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd).”

- 4.NF.5 - “Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.2 For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.”

- 4.NF.3a - “Understand a fraction a/b with a > 1 as a sum of fractions 1/b. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.”

The next relevant standard

- 5.NF.4b - “Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.”

Resources

Websites

- Khan Academy
  - “Adding fractions with unlike denominators word problem” [video] (grade 5)
  - “Subtracting fractions with unlike denominators word problem” [video] (grade 5)
  - “Adding and subtracting fractions with unlike denominators word problems” [practice] (grade 5)
  - “Adding and subtracting mixed numbers 1 (ex 1)” [video] (grade 5)
  - “Adding and subtracting mixed numbers 1 (ex 2)” [video] (grade 5)
  - “Adding and subtracting mixed number 1” [practice] (grade 5)
  - “Adding fractions with unlike denominators” [video] (grade 5)
  - “Adding fractions (ex 1)” [video] (grade 5)
  - “Adding fractions with unlike denominators” [practice] (grade 5)
  - “Subtracting fractions with unlike denominators” [video] (grade 5)
  - “Subtracting fractions with unlike denominators” [practice] (grade 5)
  - “Introduction to equivalent fractions” [video] (grade 4)
  - “Visualizing equivalent fractions” [video] (grade 4)
  - “Equivalent fraction word problem example” [video] (grade 4)
  - “Equivalent fraction word problem example 2” [video] (grade 4)
  - “Equivalent fraction word problem example 3” [video] (grade 4)
  - “Visualizing equivalent fractions” [practice] (grade 4)
  - “More on equivalent fractions” [video] (grade 4)
  - “Equivalent fractions example” [video] (grade 4)
  - “Equivalent fractions” [practice] (grade 4)
  - “Fractions cut and copy 1” [practice] (grade 4)
  - “Fractions cut and copy 1” [practice] (grade 4)
  - “Fractions word problems 1” [video] (grade 4)
  - “Fraction word problems 1” [practice] (grade 4)
  - “What fraction of spider eyes are looking at me?” [video] (grade 4)
  - “How much more piano practice?” [video] (grade 4)
  - “Adding and subtracting fractions with like denominators word problems” [practice] (grade 4)
  - “Decomposing a fraction visually” [video] (grade 4)
  - “Decomposing a mixed number” [video] (grade 4)
  - “Adding up to a fraction drag and drop example” [video] (grade 4)
  - “Decomposing fractions” [practice] (grade 4)
- Fractions review* [practice] [grade 5]
- Equivalent fractions* [practice] [grade 5]
- Reduce fractions to lowest terms* [practice] [grade 5]
- Convert between improper fractions and mixed numbers* [practice] [grade 5]
- Least common denominator* [practice] [grade 5]
- Add and subtract fractions with like denominators* [practice] [grade 5]
- Add and subtract mixed numbers with like denominators* [practice] [grade 5]
- Add fractions with unlike denominators* [practice] [grade 5]
- Subtract fractions with unlike denominators* [practice] [grade 5]
- Add 3 or more fractions with unlike denominators* [practice] [grade 5]
- Add mixed numbers with unlike denominators* [practice] [grade 5]
- Subtract mixed numbers with unlike denominators* [practice] [grade 5]
- Add and subtract fractions with like denominators* [practice] [grade 6]
- Add and subtract fractions with unlike denominators* [practice] [grade 6]
- Add and subtract mixed numbers* [practice] [grade 6]
- Add and subtract fractions* [practice] [grade 7]
- Add and subtract mixed numbers* [practice] [grade 7]

iPad Apps
- Khan Academy - Get the App [all grades]

IXL
- Get the App [all grades]
Common Core Math

Converting Between Units
Grade 5 -- 5.MD.1

"Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems."
- CCSS 5.MD.1

Expectations for Students

Students must be able to add, subtract, multiply, and divide distances.
For example: 2 feet + 2 feet = 4 feet
5 inches + 3 inches = 2 inches
15 meters x 2 = 30 meters
15 meters x 2 meters = 30 square meters
20 miles / 2 miles = 10
20 miles / 2 = 10 miles
20 square miles / 2 miles = 10 miles

Students must be able to add, subtract, multiply, and divide intervals of time.
For example: 25 seconds + 13 seconds = 38 seconds
18 minutes - 5 minutes = 8 minutes
6 minutes x 4 = 24 minutes
60 minutes / 10 = 6 minutes
60 minutes / 10 minutes = 6

Students must be able to add, subtract, multiply, and divide liquid volumes.
For example: 2 gallons = 5 gallons = 7 gallons
10 litres - 4 litres = 6 litres
3 pints x 4 = 12 pints
15 quarts / 3 = 5 quarts
15 quarts / 3 quarts = 5

Students must be able to add, subtract, multiply, and divide masses of objects.
For example: 2 kg + 7 kg = 9 kg
25 lbs - 15 lbs = 10 lbs
14 g x 3 = 42 g
16 ounces / 4 = 4 ounces
16 ounces / 4 ounces = 4

Students must be able to add, subtract, multiply, and divide money.
For example: 50 cents + 25 cents = 75 cents
$2.00 - $2.50 = 50 cents
4 dimes x 10 = 40 dimes
24 quarters / 4 = 6 quarters
24 quarters / 4 quarters = 6

Students must be able to add, subtract, multiply, and divide fractions.
For example: 3/4 + 3/4 = 6/4
5/7 - 2/7 = 3/7
1/4 x 2/3 = 2/12
4/7 / 3/4 = 4/7 x 4/3 = 16/21
Students must be able to add, subtract, multiply, and divide decimals.
For example:
1.4 + 2.7 = 4.1
5.8 - 3.5 = 2.3
1.2 x 3.0 = 3.6
4.8 / 1.2 = 4.0

Students must be able to convert between units within the same measurement "family".
For example:
1 foot = 12 inches, 1 meter = 100 centimeters
1 minute = 60 seconds, 1 hour = 60 minutes, 1 day = 24 hours, 1 week = 7 days
1 quart = 2 pints, 1 gallon = 4 quarts, 1 cup = 16 oz
1 lb = 16 ounces, 1 kilogram = 1,000 grams
$1 = 100 cents = 100 pennies = 20 nickels = 10 dimes = 4 quarters

From the Teachers
In an interview, fifth grade teachers noted that students commonly have difficulty with converting between units (for example: inches, feet, yards; centimeter, decimeter, meter).

Progression

The previous three relevant standards

4.MD.2. “Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.”

4.MD.1. “Know relative sizes of measurement units within one system of units including km, m, cm; kg, g, lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36),…”

3.MD.2. “Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.”

The next relevant standard

5.MD.5a. “Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.”
Resources

Websites

Khan Academy
- "Converting minutes to hours" [video] [grade 5]
- "Ordering metric distances" [video] [grade 5]
- "Converting centimeters to meters" [video] [grade 5]
- "Converting gallons to quarts, pints, and cups" [video] [grade 5]
- "Performing arithmetic calculations on units of volume" [video] [grade 5]
- "Unit conversion with fractions" [video] [grade 5]
- "Converting units" [practice] [grade 5]
- "Distance in the metric system" [video] [grade 4]
- "Metric units for fluid volume" [video] [grade 4]
- "Metric units for weight" [video] [grade 4]
- "US customary units for distance" [video] [grade 4]
- "US customary units for weight" [video] [grade 4]
- "US customary units for fluid volume" [video] [grade 4]
- "Examples determining reasonable units" [video] [grade 4]
- "Pounds or ounces to measure weight" [video] [grade 4]
- "Unit sense" [practice] [grade 4]
- "Converting between units of time" [video] [grade 4]
- "Converting feet to inches" [video] [grade 4]
- "Converting distances in the metric system" [video] [grade 4]
- "Converting fluid volume in US customary units" [video] [grade 4]
- "Converting weight in metric and US customary systems" [video] [grade 4]
- "Measurement units" [practice] [grade 4]

IXL
- "Choose the appropriate customary unit of measure" [practice] [grade 5]
- "Choose the appropriate metric unit of measure" [practice] [grade 5]
- "Compare and convert customary units" [practice] [grade 5]
- "Compare and convert metric units" [practice] [grade 5]
- "Compare customary units by multiplying" [practice] [grade 5]
- "Convert customary units involving fractions" [practice] [grade 5]
- "Convert mixed customary units" [practice] [grade 5]
- "Add and subtract mixed customary units" [practice] [grade 5]
- "Reasonable temperature - Celsius and Fahrenheit" [practice] [grade 5]
- "Convert time units" [practice] [grade 5]
- "Add and subtract time units" [practice] [grade 5]
- "Add and subtract money amounts" [practice] [grade 4]
- "Add, subtract, multiply, and divide money amounts" [practice] [grade 4]
- "Compare and convert customary units" [practice] [grade 4]
- "Compare and convert metric units" [practice] [grade 4]
- "Compare customary units by multiplying" [practice] [grade 4]
- "Convert mixed customary units" [practice] [grade 4]
- "Add and subtract mixed customary units" [practice] [grade 4]

iPad Apps

Khan Academy
- Get the App [all grades]

IXL
- Get the App [all grades]
Finding Volume of Irregular Figures
Grade 5 -- 5.MD.3b

"A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units."
- CCSS 5.MD.3b

Expectations for Students

Students must be able to fill (without gaps or overlaps) a regular figure using unit cubes.
For example: *Volume Shape Game** Cuboid Exploder** Volume* *Volume Using the Unit Cube*

Students must be able to partition irregular figures into a set of regular figures.
For example: An L-shaped figure can be partitioned into a set made up of two rectangular (regular) prisms [ ] and ___
A set containing two rectangular prisms can be placed together to form an L-shaped figure

Students must be able to recognize that a set of regular figures has an additive total volume.
For example: An L-shaped figure can be partitioned into a set made up of two rectangular (regular) prisms
A set containing two rectangular prisms can be placed together to form an L-shaped figure
The volume of the L-shaped figure is the same as the sum of the volumes of the two rectangular prisms in the set

From the Teachers

In an interview, fifth grade teachers noted that students commonly have difficulty with finding volume of irregular figures by partitioning the irregular figures into sets of regular figures.

Progression

The previous two relevant standards

5.MD.3b - "Recognize volume as an attribute of solid figures and understand concepts of volume measurement. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume."

3.MD.2 - "Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem."

The next relevant standard

5.MD.4 - "Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units."
Resources

Websites
Khan Academy
- “How we measure volume” [video] [grade 5]
- “Measuring volume with unit cubes” [video] [grade 5]
- “Volume with unit cubes 1” [practice] [grade 5]
- “Measuring volume as area times length” [video] [grade 5]
- “Volume of a rectangular prism or box examples” [practice] [grade 5]
- “Volume 1” [practice] [grade 5]
- “Volume word problem example” [video] [grade 5]
- “Volume word problems” [practice] [grade 5]
- “Categorize quadrilaterals” [practice] [grade 3]
- “Cutting shapes into equal parts” [practice] [grade 3]
- “Volume of a rectangular prism with fractional dimensions” [video] [grade 6]
- “Volume with fractions” [practice] [grade 6]
- “Volume of a rectangular prism with fractional cubes” [video] [grade 6]
- “Volume with unit cubes 2” [practice] [grade 6]
- “Volume of marbles in a fish tank” [video] [grade 6]
- “Volume word problems with fractions” [practice] [grade 6]

IXL
- “Volume and surface area of similar solids” [practice] [grade 8]
- “Volume of cubes and rectangular prisms” [practice] [grade 6]
- “Volume and surface area of triangular prisms” [practice] [grade 6]
- “Volume and surface area of cylinders” [practice] [grade 6]
- “Volume” [practice] [grade 4]
- “Volume” [practice] [grade 3]

Other
- “Finding Volume with Unit Cubes” [PDF] worksheets with answer keys [grade 5]

iPad Apps
Khan Academy
- Get the App [all grades]
IXL
- Get the App [all grades]
Finding Volume of Irregular Figures
Grade 5 -- 5.MD.4

*Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.*
- CCSS 5.MD.4

Expectations for Students

Students must be able to fill (without gaps or overlaps) a regular figure using unit cubes.
For example: *Volume Shape Game* *Cuboid Explode* *Volume* *Volume Using the Unit Cube*

Students must be able to partition irregular figures into a set of regular figures.
For example: An L-shape figure can be partitioned into a set made up of two rectangular (regular) prisms [ ___ and ___ ]
A set containing two rectangular prisms can be placed together to form an L-shaped figure

Students must be able to recognise that a set of regular figures has an additive total volume.
For example: An L-shaped figure can be partitioned into a set made up of two rectangular (regular) prisms
A set containing two rectangular prisms can be placed together to form an L-shaped figure
The volume of the L-shaped figure is the same as the sum of the volumes of the two rectangular prisms in the set

Students must be able to label unit cubes appropriately.
For example: A unit cube with side length 1 cm has a volume of 1 cubic cm
A unit cube with side length 1 in has a volume of 1 cubic in
A unit cube with side length 1 ft has a volume of 1 cubic ft
A unit cube with side length 1 "enter unit here" has a volume of 1 cubic "enter unit here"

From the Teachers

In an interview, fifth grade teachers noted that students commonly have difficulty with finding volume of irregular figures by partitioning the irregular figures into sets of regular figures.

Progression

5.MD.3b - *Recognize volume as an attribute of solid figures and understand concepts of volume measurement. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.*

5.MD.3c - *Recognize volume as an attribute of solid figures and understand concepts of volume measurement. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.*
The next relevant standard 5.MD.5a: “Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.”

Resources

Websites
- Khan Academy
  - “How we measure volume” [video] [grade 5]
  - “Measuring volume with unit cubes” [video] [grade 5]
  - “Volume with unit cubes II” [practice] [grade 5]
  - “Measuring volume as area times length” [video] [grade 5]
  - “Volume of a rectangular prism or box examples” [practice] [grade 5]
  - “Volume I” [practice] [grade 5]
  - “Volume word problem example” [video] [grade 5]
  - “Volume word problems” [practice] [grade 5]
  - “Categorize quadrilaterals” [practice] [grade 3]
  - “Cutting shapes into equal parts” [practice] [grade 3]
  - “Volume of a rectangular prism with fractional dimensions” [video] [grade 6]
  - “Volume with fractions” [practice] [grade 6]
  - “Volume of a rectangular prism with fractional cubes” [video] [grade 6]
  - “Volume with unit cubes II” [practice] [grade 6]
  - “Volume of marbles in a fish tank” [video] [grade 6]
  - “Volume word problems with fractions” [practice] [grade 6]

IXL
- “Volume and surface area of similar solids” [practice] [grade 8]
- “Volume of cubes and rectangular prisms” [practice] [grade 6]
- “Volume and surface area of triangular prisms” [practice] [grade 6]
- “Volume and surface area of cylinders” [practice] [grade 6]
- “Volume” [practice] [grade 4]
- “Volume” [practice] [grade 3]

Other
- “Finding Volume with Unit Cubes” [PDF; worksheets with answer keys] [grade 5]

iPad Apps
- Khan Academy
  - Get the App [all grades]
- IXL
  - Get the App [all grades]
Common Core Math

Graphing on the Four-Quadrant Coordinate Plane
Grade 5 -- 5.G.2

"Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation."
- CCSS 5.G.2

Expectations for Students
Students must be able to graph any coordinate point onto the coordinate plane (within the first quadrant).
For example: The coordinate pair (4, 5) has an X-value of 4 and a Y-value of 5
To graph, travel a distance of 4 along the X-axis, then a distance of 5 along the Y-axis
The movement to plot the coordinate point (4, 5) is left 4 then up 5

Students must be able to interpret a point on the coordinate plane based on the given context.
For example: For a graph with Time in seconds on the X-axis and Distance Traveled in feet on the Y-axis
The coordinate point (4, 5) can be interpreted as "At 4 seconds, the distance traveled is 5"

From the Teachers
In an interview, fifth grade teachers noted that students commonly have difficulty with graphing (in any quadrant) on the four-quadrant coordinate plane. Students often confuse the X-coordinate with the Y-coordinate.

Progression

The previous three relevant standards

5.G.1 - "Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and y-coordinate, y-axis and x-coordinate)."

The next relevant standard

6.G.3 - "Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems."

6.NS.6b - "Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes."

-
6.NS.B: “Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.”

Resources

Websites
- Khan Academy
  - "Graphing points exercise" [video] [grade 5]
  - "Graphing points" [practice] [grade 5]
  - "Blocks: Naomi walks from school" [video] [grade 5]
  - "Coordinate plane word problems in the first quadrant" [practice] [grade 5]
  - "The coordinate plane" [video] [grade 6]
  - "Plot ordered pairs" [video] [grade 6]
  - "Quadrants of coordinate plane" [video] [grade 6]
  - "Graphing points and naming quadrants" [practice] [grade 6]
  - "Points on the coordinate plane" [video] [grade 6]
  - "Points on the coordinate plane" [practice] [grade 6]
  - "Coordinate plane word problems exercise" [video] [grade 6]
  - "Coordinate plane word problems in all four quadrants" [practice] [grade 6]
- IXL
  - "Coordinate graphs review - whole numbers only" [practice] [grade 5]
  - "Coordinate graphs with decimals and negative numbers" [practice] [grade 5]
  - "Graph points on a coordinate plane" [practice] [grade 5]
  - "Coordinate graphs as maps" [practice] [grade 5]
  - "Relative coordinates: follow directions" [practice] [grade 5]
  - "Quadrants" [practice] [grade 5]
  - "Coordinate graphs review" [practice] [grade 4]
  - "Graph points on a coordinate plane" [practice] [grade 4]
  - "Relative coordinates" [practice] [grade 4]
  - "Coordinate graphs as maps" [practice] [grade 4]
  - "Coordinate graphs review" [practice] [grade 6]
  - "Graph points on a coordinate plane" [practice] [grade 6]
  - "Distance between two points" [practice] [grade 6]
  - "Find points on a function graph" [practice] [grade 6]

iPad Apps
- Khan Academy
  - Get the App [all grades]
- IXL
  - Get the App [all grades]
Common Core Math

Naming Shapes/Figures based on Attributes
Grade 5 -- 5.G.3

*Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

- CCSS 5.G.3

Expectations for Students

Students must be able to recognize attributes of shapes/figures.
For example:
- Quadrilateral, rectangular, four-sided, all equal sides, and 90-degree angles are all attributes of a square
- Quadrilateral, rectangular, four-sided, and 90-degree angles are all attributes of a rectangle

Students must be able to recognize categories and their subcategories.
For example:
- The category “Rectangles” has a subcategory “Squares”
  All squares are rectangles
  Not all rectangles are squares

Students must be able to recognize that an attribute of a category is also an attribute of all of its subcategories.
For example:
- The category “Rectangles” has a subcategory “Squares”
  All rectangles have four sides
  All squares are rectangles, so all squares have four sides
  All squares have four equal sides
  Not all rectangles are squares, so not all rectangles have four equal sides

From the Teachers

In an interview, fifth grade teachers noted that students commonly have difficulty with naming shapes/figures based on given attributes.
Progression

The previous three relevant standards

5.6.4 - “Classify two-dimensional figures in a hierarchy based on properties.”

4.0.2 - “Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.”

3.6.1 - “Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.”

The next relevant standard

7.6.2 - “Draw freehand, with ruler and protractor, and with technology geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.”

Resources

Websites

Khan Academy - * “Categorize quadrilaterals” [practice] [grade 3]*
- *“Quadrilateral overview” [video] [grade 4]*
- *“Quadrilateral properties” [video] [grade 4]*
- *“Kites as a mathematical shape” [video] [grade 4]*
- *“Quadrilateral types exercise” [video] [grade 4]*
- *“Quadrilateral types” [practice] [grade 4]*
- *“Scalene, isosceles, equilateral: acute, right, and obtuse triangles” [video] [grade 4]*
- *“Triangle types” [practice] [grade 4]*
- *“Classifying triangles” [video] [grade 4]*
- *“Recognizing triangles” [practice] [grade 4]*
- *“Examples with perpendicular lines and right, obtuse, acute triangles” [video] [grade 4]*
- *“Classifying shapes by line and angle types” [practice] [grade 4]*
- *“Classifying shapes” [video] [grade 5]*
- *“Properties of shapes” [practice] [grade 5]*

IXL

- *“Is it a polygon?” [practice] [grade 3]*
- *“Triangles: equilateral, isosceles, and scalene” [practice] [grade 3]*
- *“Triangles: acute, right, and obtuse” [practice] [grade 3]*
- *“Classify quadrilaterals” [practice] [grade 3]*
- *“Which 2-dimensional shape is being described?” [practice] [grade 4]*
- *“Classify quadrilaterals” [practice] [grade 4]*
- *“Number of sides in polygons” [practice] [grade 4]*
- *“Types of triangles” [practice] [grade 5]*
- *“Number of sides in polygon” [practice] [grade 5]*
- *“Which figure is being described?” [practice] [grade 5]*
- *“Classify quadrilaterals” [practice] [grade 5]*
- *“Classify quadrilaterals” [practice] [grade 6]*
- *“Classify triangles” [practice] [grade 7]*
- *“Classify quadrilaterals” [practice] [grade 7]*
- *“Identify and classify polygon” [practice] [grade 7]*

iPad Apps

Khan Academy - Get the App [all grades]
IXL - Get the App [all grades]
Common Core Math

Links to online math resources

Links to iPad applications

“We are here for knowledge. Our enemy is thoughtlessness. This is philosophy.” — Damon Horowitz

Most resource links on the Sticking Points pages are from the Khan Academy and IXL websites.

Photo used under Creative Commons from William Hook
POWERED BY weebly
Common Core Math

Online Resources

- Khan Academy: http://www.khanacademy.org
- IXL Learning: http://www.ixl.com/math/
- Common Core Sheets: http://www.commoncoresheets.com
- Interactive Sites for ED: http://interactivesites.weebly.com/math.html
- LearnZillion: http://learnzillion.com
- Math Drills: http://www.math-drills.com
- Math-Play: http://www.math-play.com
- PBS Kids Math Games: http://pbskids.org/games/math/
- Soft Schools: http://www.softschools.com/math/games/

Photo used under Creative Commons from chiparps
POWERED BY weebly
# iPad Resources

<table>
<thead>
<tr>
<th>App Name</th>
<th>Description from App Store</th>
<th>Link to App Store</th>
<th>Price*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Khan Academy**</td>
<td>&quot;Khan Academy allows you to learn almost anything for free... View Khan Academy's complete library of over 4,000 videos&quot;</td>
<td><a href="https://itunes.apple.com/us/app/khan-academy/id4408863705?mt=8">https://itunes.apple.com/us/app/khan-academy/id4408863705?mt=8</a></td>
<td>FREE</td>
</tr>
<tr>
<td>Abacus for Kids</td>
<td>&quot;This application includes the Basic Training... Abacus Formulas... [and] Abacus Tests.&quot;</td>
<td><a href="https://itunes.apple.com/us/app/abacus-for-kids/id6462646400?mt=8">https://itunes.apple.com/us/app/abacus-for-kids/id6462646400?mt=8</a></td>
<td>FREE</td>
</tr>
<tr>
<td>Action Grapher Algebra</td>
<td>&quot;Challenge[es] the student to explore how the information from each axis influences the graph&quot;</td>
<td><a href="https://itunes.apple.com/us/app/action-grapher-algebra/id6215742517?mt=8">https://itunes.apple.com/us/app/action-grapher-algebra/id6215742517?mt=8</a></td>
<td>FREE</td>
</tr>
<tr>
<td>App Name</td>
<td>Description</td>
<td>Price</td>
<td>URL</td>
</tr>
<tr>
<td>----------------------------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>--------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>Algebra Touch**</td>
<td>&quot;Refresh your (algebra) skills using touch-based techniques built from the</td>
<td>$2.99</td>
<td><a href="https://itunes.apple.com/us/app/algebra-touch/id38435426?mt=8">Link</a></td>
</tr>
<tr>
<td>Cover Up Math</td>
<td>&quot;Given an equation that is &quot;messy&quot; with fractions, exponents, square roots,</td>
<td>FREE</td>
<td><a href="https://itunes.apple.com/us/app/cover-up/id54176466?mt=8">Link</a></td>
</tr>
<tr>
<td>Diamond Factor</td>
<td>&quot;Learn how to factor trinomials of various difficulties&quot;</td>
<td>FREE</td>
<td><a href="https://itunes.apple.com/us/app/diamond-factor/id54640954?mt=8">Link</a></td>
</tr>
<tr>
<td>Dice Tracker</td>
<td>&quot;This app creates and tracks dice rolls... Watch your roll distribution</td>
<td>FREE</td>
<td><a href="https://itunes.apple.com/us/app/dice-tracker/id598518166?mt=8">Link</a></td>
</tr>
<tr>
<td>Equivalent Fractions**</td>
<td>&quot;Create equivalent fractions by dividing and shading squares or circles, and</td>
<td>FREE</td>
<td><a href="https://itunes.apple.com/us/app/equivalent-fractions/id5480510?mt=8">Link</a></td>
</tr>
<tr>
<td>Function Mystery Machine**</td>
<td>&quot;Choose a level or go head-to-head with a friend as you try to guess the</td>
<td>FREE</td>
<td><a href="https://itunes.apple.com/us/app/function-mystery-machine/id64580201?mt=8">Link</a></td>
</tr>
<tr>
<td>Linear Model</td>
<td>&quot;The app allows you to enter a linear equation to be displayed on the graph,</td>
<td>FREE</td>
<td><a href="https://itunes.apple.com/us/app/linear-model/id4864418?mt=8">Link</a></td>
</tr>
<tr>
<td>Lion Grapher</td>
<td>&quot;See how many lions you can get with linear equations before the time runs out!&quot;</td>
<td>FREE</td>
<td><a href="https://itunes.apple.com/us/app/lion-grapher/id53178738?mt=8">Link</a></td>
</tr>
<tr>
<td>Math Flyer</td>
<td>&quot;The best tool to use to really understand the meaning of the parts of a mathematical expression&quot;</td>
<td>$0.99</td>
<td><a href="https://itunes.apple.com/us/app/math-flyer/id44955952?mt=8">Link</a></td>
</tr>
<tr>
<td>Math Ref Free**</td>
<td>&quot;This app gives you just a sample (over 700) of the over 1,400 helpful formulas, figures, tips, and examples that are included in the full version of Math Ref&quot;</td>
<td>FREE</td>
<td><a href="https://itunes.apple.com/us/app/math-ref-free/id3127535?mt=8">Link</a></td>
</tr>
<tr>
<td>App Name</td>
<td>Description</td>
<td>URL</td>
<td>Price</td>
</tr>
<tr>
<td>-----------------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>Point Plotter</td>
<td>&quot;A game that tests your knowledge of the 2D coordinate system and your ability to recognize patterns&quot;</td>
<td><a href="https://itunes.apple.com/us/app/point-plotter/id499972563?mt=8">https://itunes.apple.com/us/app/point-plotter/id499972563?mt=8</a></td>
<td>FREE</td>
</tr>
<tr>
<td>Tappy**</td>
<td>&quot;Tappy is a puzzle where all tiles have to be tapped in ascending sequence (including) 'Numbers', 'Digits &amp; alphabet', 'Fractions', 'Inches', 'Time', 'Roman numerals', 'Binary', 'Octal', 'Hexadecimal'&quot;</td>
<td><a href="https://itunes.apple.com/us/app/tappy/id587363291?mt=8">https://itunes.apple.com/us/app/tappy/id587363291?mt=8</a></td>
<td>FREE</td>
</tr>
</tbody>
</table>

*Prices are as of May 2014, and may vary.
**Recommended by CommonCoreMathParents.weebly.com
Dear Parents...

Teachers appreciate your help in the classroom,

and now they need it from home, too.

As parents, you are often recruited to chaperone field trips and school dances. You are invited into the classroom as aides during arts and crafts. As parents, you provide your student with the supplies they need to succeed, and often donate more for general classroom use.

Now, teachers are asking for your help in the context of your student’s education as well.

Fourth and fifth grade teachers from a Title 1 elementary school were interviewed regarding the new Common Core State Standards for math in Oregon. During the interviews, the teachers were asked what they needed from parents at home. The following is an excerpt from the Western Oregon University Undergraduate Honors Thesis Common Core Math: A Guide by Stephanie Eaton (bold text added here for emphasis and easier reading):
In both grades, all teachers (A-E) requested that students be held accountable for learning their basic math facts to mastery. Teacher D stated that “rote recognition...would be a huge time saver, and [the student] would be so much more successful.” The fifth grade group requested that students practice **addition and multiplication facts** at home, enough to have “instant recall” of the basic math facts (Teacher E). Both groups noted that memorization of the basic math facts are not a part of their standards under Common Core, but that the “instant recall” would aid students in many areas which are a part of the Common Core standards.

Three primary skills were observed by both groups to be topics which would best be practiced and mastered at home once introduced at school in the earlier grades: **telling time, counting/exchanging money, and measuring with a ruler.** None of the three skills are part of the standards for grades four or five, but the teachers noted that time is still spent on “catching students up”. Teacher C observed: “getting [students] to where they always get the clock, they always are able to add up the coins...I feel that that would help because it would help with our decimals when we do the money...and being able to use a clock and tell time.”

The third topic raised by both groups was **learning language through conversation.** Teacher B noted that students are “missing their shared language experience”, which Teacher C added might be a result of the culture in the community. The fifth grade group requested that parents take the time to **generate student respect for both language and math** through **mathematical conversations** at home, an example being fractions and measurement through **measuring cups while cooking,** which the fourth grade group suggested. Teacher C also suggested that family conversation “develops oral language, (and) develops concepts that [students] are missing”.

The fourth topic, raised by Teachers D and E, was **student accountability for homework.** The group agreed that they needed support from parents to keep the students themselves accountable for completing and returning their homework. Teacher E expressed that when parents take the time to **show their concern for student homework,** the student sees “that the parents feel that education is important. But if [parents] never look at their [student’s] work, the kids get the message that the parents don’t care.”

The following is a list of items which the interviewed teachers felt would be helpful to have at home as physical resources:

- Clocks that can be manipulated — with minutes
- Coins — Real or fake
- Cubes — Manipulatives, including unifix cubes
- Flash Cards — Addition, Subtraction, Multiplication, and Division
- Fraction pieces, bars, and shapes
- Geoboards — for area and perimeter
- Hundreds Boards — number charts
- Number Lines

---

Appendix
Interview Questions

I. What grade(s) are you currently teaching?

II. Do you have experience teaching any other grades?
   A. If yes, which grade(s)?
   B. If yes, how many years in each grade?

III. Are you familiar with Oregon’s new Common Core State Standards for Math?

IV. Has your school adopted the new Common Core State Standards for Math?
   A. If no, when does the school plan to adopt them, if at all?
   B. If yes, have you integrated them into your classroom?

V. Are there any specific standards or cluster of standards which you feel ___ grade
   students are not meeting as a general group? I.e., are there any specific concepts or
   areas which generally pose as problem areas for your ___ graders?

For each problem area:

   • When should the problem area have been introduced?
   • When should it have been/be mastered?
   • To the best of your knowledge, what is keeping the students from mastering the
     content?
   • Are the students coming in with the background knowledge required for the
     problem area?
     • If not, what are they missing?
• Are there any strategies that parents and guardians at home can use to help their student succeed in this area?

VI. After reviewing the Oregon Common Core State Standards (OCCSS) Grade ___ Overview from the Oregon Department of Education’s document “Common Core State Standards for Mathematics”:

A. Are there any general standards which your students routinely struggle with?
   1. If yes, can you expand on the problem area?

VII. Based on your experience, which topics take the longest time to cover? Why might this be?

VIII. Are there any topics which you are routinely having to reteach or review in depth?

A. Are these topics new to the students this year, or should they have been (or have they been) taught in previous years?

IX. What is your best guess as to why students are having such difficulty in these areas?

X. Where does each problem fall on the timeline of each school year?

XI. Would you say that content retention over the breaks is a problem? I.e., are students retaining the appropriate type and/or amount of content over the summer/winter/spring break?

A. If so, could you suggest any activities/etc. that students could do during breaks to catch up/maintain/exceed the standards?

B. How can we make students’ experiences in math more efficient/effective/etc.?
Sticking Points Progression

The following appendix of Sticking Point Standards (SPS) for grades four and five shows the progression related to each SPS. The previous few relevant standards, along with the next one or two relevant standards are given. In this way, the SPS (in oblique) is placed in context of the progression of skills through which students will be led. Each standard was copied word for word from the Common Core State Standards Initiative website.

Grade 4 Sticking Point Standards

4.OA.2 — Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

The previous relevant standards 4.OA.1 - "Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations."
3.OA.6 - “Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.”

3.OA.3 - “Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.”

The next relevant standard is 5.OA.2 - “Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.”

4.OA.3 — Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
The previous relevant standards

4.NBT.6 - “Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.”

4.NBT.5 - “Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.”

3.OA.7 - "Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers."

The next relevant standard

5.NBT.5 - “Fluently multiply multi-digit whole numbers using the standard algorithm.”
4.OA.4 — Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

The previous relevant standards

4.OA.1 - "Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 \times 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations."

3.OA.7 - "Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 \times 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers."
3.OA.5 - "Apply properties of operations as strategies to multiply and divide. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find $8 \times 7$ as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)"

The next relevant standard 6.NS.4 - "Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express $36 + 8$ as $4 (9 + 2)$.

4.NBT.3 — Use place value understanding to round multi-digit whole numbers to any place.
The previous relevant standards

4.NBT.2 - “Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.”

4.NBT.1 - “Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.”

The next relevant standard

4.NF.7 - “Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.”

5.NBT.3b - “Read, write, and compare decimals to thousandths. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.”
4.NBT.4 — *Fluently add and subtract multi-digit whole numbers using the standard algorithm.*

The previous relevant standards:

3.NBT.2 - “Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.”

2.NBT.5 - “Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.”

The next relevant standard:

4.NBT.5 - “Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.”

4.NBT.6 — *Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.*
The previous relevant standards

4.NBT.5 - “Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.”

The next relevant standard

4.OA.3 - “Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.”

5.NBT.5 - “Fluently multiply multi-digit whole numbers using the standard algorithm.”

4.NF.2 — Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions
refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

The previous relevant standards

3.NF.3b - “Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.”

3.NF.2b - “Understand a fraction as a number on the number line; represent fractions on a number line diagram. Represent a fraction $a/b$ on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size $a/b$ and that its endpoint locates the number $a/b$ on the number line.”

The next relevant standard

5.NF.1 - “Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$)”
4.MD.2 — Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.MD.1 - “Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...”
3.MD.2 - “Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.”

The next relevant standard 5.MD.1 - “Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.”

Grade 5 Sticking Point Standards

5.OA.1 — Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
The previous relevant standards 3.OA.8 - “Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.”

The next relevant standard 5.OA.2 - “Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.”

6.EE.2a - “Write, read, and evaluate expressions in which letters stand for numbers. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract $y$ from 5" as $5 - y$.”

5.NBT.2 — Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when
a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

The previous relevant standards

5.NBT.1 - “Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.”

5.NBT.3b - “Read, write, and compare decimals to thousandths. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.”

4.NF.7 - “Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.”

The next relevant standard

5.NBT.4 - “Use place value understanding to round decimals to any place.”

6.EE.1 - “Write and evaluate numerical expressions involving whole-number exponents.”
5.NBT.3a — Read, write, and compare decimals to thousandths. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g.,

\[ 347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000). \]

The previous relevant standards

5.NBT.1 - “Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.”

4.NBT.3 - “Use place value understanding to round multi-digit whole numbers to any place.”

The next relevant standard

5.NBT.4 - “Use place value understanding to round decimals to any place.”

6.NS.3 - “Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.”

5.NBT.3b — Read, write, and compare decimals to thousandths. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
The previous relevant standards:

4.NF.7 - “Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.”

4.NF.6 - “Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.”

The next relevant standard:

5.NBT.2 - “Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.”

5.NBT.4 - “Use place value understanding to round decimals to any place.”

5.NBT.4 — Use place value understanding to round decimals to any place.
The previous relevant standards 5.NBT.3a - “Read, write, and compare decimals to thousandths. Read and write decimals to thousandths using base-
ten numerals, number names, and expanded form, e.g.,
347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000).”

4.NBT.3 - “Use place value understanding to round multi-digit whole numbers to any place.”

The next relevant standard 5.NBT.7 - “Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.”

6.NS.3 - “Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.”

5.NBT.6 — Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
5.NF.1 — Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.
The previous relevant standards

4.NF.5 - “Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.”

4.NF.3a - “Understand a fraction \( \frac{a}{b} \) with \( a > 1 \) as a sum of fractions \( \frac{1}{b} \). Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.”

3.NF.3a - “Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.”
The next relevant standard 5.NF.2 - “Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.”

5.NF.2 — Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

The previous relevant standards 5.NF.1 - “Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)”
4.NF.5 - “Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.”

4.NF.3a - “Understand a fraction $a/b$ with $a > 1$ as a sum of fractions $1/b$. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.”

The next relevant standard 5.NF.4b - “Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.”

5.MD.1 — Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.
The previous relevant standards

4.MD.2 - “Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.”

4.MD.1 - “Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...”
5.MD.3b — Recognize volume as an attribute of solid figures and understand concepts of volume measurement. A solid figure which can be packed without gaps or overlaps using \( n \) unit cubes is said to have a volume of \( n \) cubic units.

3.MD.2 - “Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.”

The next relevant standard is 5.MD.5a - “Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.”
The previous relevant standards 5.MD.3a - “Recognize volume as an attribute of solid figures and understand concepts of volume measurement. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.”

3.MD.2 - “Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.”

The next relevant standard 5.MD.4 - “Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.”

5.MD.4 — Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

The previous relevant standards 5.MD.3b - “Recognize volume as an attribute of solid figures and understand concepts of volume measurement. A solid figure which can be packed without gaps or overlaps using \( n \) unit cubes is said to have a volume of \( n \) cubic units.”
**5.MD.3a** - “Recognize volume as an attribute of solid figures and understand concepts of volume measurement. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.”

The next relevant standard is **5.MD.5a** - “Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.”

**5.G.2** — *Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.*
The previous relevant standards

**5.G.1** - “Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., \(x\)-axis and \(x\)-coordinate, \(y\)-axis and \(y\)-coordinate).”

The next relevant standard

**6.G.3** - “Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.”
6.NS.6b - “Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.”

6.NS.8 - “Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.”

5.G.3 — Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

The previous relevant standards 5.G.4 - “Classify two-dimensional figures in a hierarchy based on properties.”
4.G.2 - “Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.”

3.G.1 - “Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.”

The next relevant standard

7.G.2 - “Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.”
References


FocalPointsOct07.pdf


