6-2022

How to Teach Math More Effectively and Efficiently: Engaging Students In the World of Math

Daniel E. Holmberg

How to Teach Math More Effectively and Efficiently

Engaging Students In the World of Math

By

Daniel E. Holmberg

An Honors Thesis Submitted in Partial Fulfillment of the

Requirements for Graduation from the

Western Oregon University Honors Program

Dr. Cheryl Beaver,

Thesis Advisor

Dr. Gavin Keulks,

Honors Program Director

June 2022

Acknowledgements

I would like to acknowledge my thesis advisor Dr. Cheryl Beaver for her guidance, advice, and encouragement throughout this project. She has been a wonderful advisor and has pushed me to work very hard and get a very nice final product.

I would also like to acknowledge the Honors Director, Gavin Keulks, for his guidance throughout the whole process of this large project. From helping me brainstorm ideas to helping with the finishing touches, he has been an amazing help.

Table of Contents

Abstract

The United States is struggling with teaching mathematics in school. Students don't see the connection between math and the real world. Students are viewing math as a rigid set of rules instead of as a creative and exciting subject to explore. A few of the problems that are in traditional classrooms are: students having a fear of being publicly embarrassed, students memorizing methods instead of truly understanding the math , math problems not being relevant to students, and students not being engaged during class. A few methods and teaching strategies that can help solve these problems include: Gradual Release, Open Middle problems, and the 5 Practices of Discussion. By applying these strategies and tools to everyday math classrooms, math can become the intriguing and creative subject that it is meant to be.

<u>What's the Problem?</u>

In recent studies it has been shown that the U.S. is still in the middle of the pack when it comes to mathematics. From the one of the most recent Programme for International Student Assessment in 2015, where 15-year-olds from around the world are tested, the U.S. placed 38th out of 71 countries (DeSilver, D., 2017). The United States has potential to do much better than this 38th place ranking. The important questions are, why are we struggling, and what strategies are out there that can improve our math education systems?

There are many reasons that students today may struggle in math. One of the reasons is the fear of being embarrassed in math class for getting an answer incorrect or asking a question that many of their classmates may find unintelligent. The fear of being embarrassed in a math classroom causes many problems and can be rooted from several different causes. One common cause is what is called a fixed mindset. A fixed mindset is"the belief that intelligence is a fixed trait that cannot be changed" (Baker, 2017, p.2). Students decide very early in a class who is "smart" and who is "not smart". It has also been found that students who are called smart by their teacher gain a fear of losing this title of being "smart", so they are nervous to try harder problems and voice their thoughts (Boaler, J., 2009, p. 64). This problem comes from the misconception that when students struggle with something, it implies they are not good at it. Students often believe that in order to be smart, everything must be easy to you. Therefore, if you struggle, then you are not smart (Baker, 2017, p.3). A major issue with this is that since students are afraid to speak their minds in class, they are unable to see the potential in themselves. They also don't ask questions that not only would help their understanding, but could possibly help the understanding of the rest of the class.

Because students have a fixed mindset caused by this fear, they aren't able to develop what is called a growth mindset. A growth mindset is when a student believes that with hard work and persistence, they are capable of accomplishing a task or learning a concept. A growth mindset puts emphasis on asking questions and trying to understand what you are learning. Students with this mindset focus on what they can learn and achieve when they put forth effort instead of focusing only on their weaknesses (Baker, 2017, p.3).

One of the keys to fighting this problem is making the students feel safe to make educated guesses or ask questions during class. "The teacher who takes time to listen intently while a student asks a question, and responds with a willingness to explain, will create an atmosphere in which students feel at ease asking questions." (Morris, J., 1981, p. 414). When a teacher responds respectfully and in an appreciative manner to a question, the student will feel encouraged to ask more questions. When students take risks in class it is important to provide positive reinforcement especially if their risks involve providing a wrong answer. When a student is praised for trying even if they got an answer wrong, more students will feel comfortable to take risks. This causes them to realize that along with not getting criticized for saying a wrong answer, they will also be encouraged and shown appreciation for taking a risk. Another way to make students feel safe is to not isolate one student to give an answer or solve a problem in front of the class. Instead, have the students work together (Morris, J., 1981, p. 416). If a student doesn't know an answer, allow them to get help from a friend. This lessens the pressure for a student to know the answer because they know they can get support from their peers if needed.

Students are also dealing with the struggle of solving problems that are not believable or relevant. These problems use real life concepts, but in numbers that aren't realistic. Students aren't able to look at these problems through the lense of common sense because they are proposed in a make-believe manner (Boaler, J., 2009, p. 50). These math problems that are being used talk about people painting fences nonstop for hours at a constant pace, or people buying dozens of watermelons for seemingly no reason. These types of math problems mean nothing to students because they mean nothing to the real world. Hilary Rose, a sociologist who as a child was considered a "math wunderkind", was asked about doing these types of problems when she was in school. She stated that "the price I paid was to lose my sense of confidence that school math and everyday math were part of one world" (Boaler, J., 2009, p. 54). It's important to teach students that math is a subject that helps make sense of the world and isn't simply impractical imagination.

Along with trying to make math relevant to students, there has also been a difficulty with getting students engaged in the learning process. Engaging students in math class has been an issue for a few reasons. One of the main reasons is that students simply think they are there to learn methods, so they try to focus on key words and engage as little as possible. In many classrooms students are simply lectured to, but instead we need to "engage students as active and capable learners" (Boaler, J., 2016, p. 2). One of the best ways to increase engagement is to invite students to share their own mathematical thoughts into the lesson. Instead of students feeling like they are simply being taught math concepts it is better for them to feel like they are discovering these concepts. It makes them feel more involved in their learning (Boaler, J., 2009). When they feel more involved their motivation generally goes up and they gain interest in what they are learning.

On top of these struggles many students are finding that they don't have enough time in class to practice math. In many classrooms today, students are simply left with problems to do for homework, and they haven't even had time to know that they really understand the material or not. If they realize they don't understand the material they are left helpless at home in many cases and without help. Students need more time to develop questions that they can clarify before they leave.

A final aspect of math education that is often getting overlooked is getting students to actually understand the math itself instead of just memorizing it and trusting in the process that students are being taught. Something that is growing in emphasis is teaching math students to ask effective questions and justify the claims that they make in math. There is also a higher emphasis on helping students develop claims and solutions based upon finding relevant information (Boaler, J., 2016, p. 3). Memorizing has proven to be one of the least effective ways to study math. Students may remember the formula, but the concept itself will not be compressed in their minds. Since math revolves around building new ideas upon previous ones, the lack of these compressed methods and understanding causes students to struggle even more the further they get into their math education (Boaler, J., 2009).

By simply memorizing algorithms and procedures, students will also have difficulty if problems are any different from the example shown to them by their teacher. In many classrooms, a student will simply look at the example the teacher provided on the board and memorize the process the teacher used. When a student goes to a more difficult problem, they are unable to solve it because they can't adjust to the new aspects of the problem. They do not understand the basic principles of the concept enough to try different strategies to attempt the new problem (Morris, J., 1981, p. 414).

When students memorize math techniques, they miss out on an opportunity to develop their critical thinking. Math is an amazing way for students to develop this skill. However, unless math is taught in a way that requires this critical thinking in the problem solving process, students will continue to lack in their ability to have flexible thinking and problem solving skills that will benefit their future careers.

One of the main causes of this issue is the use of ineffective problems. Many problems that are being used are lacking in two major categories. The first is helping students in developing a true understanding of the concepts. The second is, giving teachers an accurate depiction of how well students are understanding the material that is being taught (Kaplinsky, R., 2020, p. 9). Students merely compute answers without truly understanding why the technique works. Because of this, when students are tested, they appear to have full understanding of the concept. Instead they only understand how to plug in numbers into a formula.

Gradual Release

One technique that can address these problems is Gradual Release. This teaching strategy puts an emphasis on making the transition between a student's learning and applying what the student has learned. In this process there are four focuses. These include: focused instruction, guided instruction, collaborative learning, and independent learning. It starts with the teacher having all the responsibility in doing the problem, and at the end the students are doing problems individually (Fisher, D. and Frey, N., 2014).

This method of teaching is different from the type of teaching many students encounter in their mathematical careers. In many cases students are simply lectured to and handed homework. In other classes after the lecture, students are given different problems to practice on their own. There is little transition from watching how the problem is done and having the student solve problems on their own. There is also limited interaction between the students and the learning process. Gradual Release encourages more input from the students and allows them to become more involved with the lesson.

The focused instruction stage is for establishing the purpose of the lesson and demonstrating the new material through examples of the concepts and strategies. One major way to establish purpose is to list key terms for them to focus on, and list the learning objectives they are going to work on that day. For demonstrating the new material, there are several parts. First, name the learning objective and explain its purpose. Describe when the concepts or skills associated with the learning objective are used, compare them to past learning, and demonstrate an example. After an example, assess the use of the skill.

A very important aspect of the focused instruction stage is determining specific learning objectives that you want to cover during the lesson. Your learning objectives are based upon the material that you are going to cover. For example, you may cover the idea of slope. A learning objective for slope may be that you want your students to be able to determine the slope of a graphed line. It is very important to make a learning objective specific. Instead of saying the students will learn the Pythagorean Theorem, say something like "students will recognize that the area of the square built on the hypotenuse of a right triangle is equal to the sum of the areas of the squares built on the legs and will conjecture that $c^2 = a^2 + b^2$" (Smith, M. S., & Stein, M. K., 2018, p. 14). By making sure your objectives are specific, it allows you to focus on how they connect to each other and to the mathematical concepts you are trying to teach (Smith, M. S., & Stein, M. K., 2018). It also helps you compare and connect it to past learnings. The use of specific learning objectives and the listing of key terms helps students understand what the goals of the lesson are. This stage helps students get an idea of terms to focus on and aspects of the next problem to look for. This is important because "in the absence of explicit attention to the lesson's purpose, students will not see the connection between the activities they are completing and the reasons why they are learning it." (Fisher, D. and Frey, N., 2014 p. 24). This stage also sets the foundation for the lesson as a whole. It gives the students a glimpse of what they are going to learn, so that when they attempt it as a class in the next stage, they are not completely in the dark.

The guided instruction stage is when the teacher goes through an example with the class. During this stage, the class is discussing together how to solve the problem. Students are suggesting ideas, and providing explanation and reasoning for why their ideas may work. As a class they decide which step to take next and either a student or the teacher writes on the board the problem step for step while the class explains what to write. The process of how the students decide what to do next involves students offering different ideas on possible strategies based upon what they learned during the focused instruction stage. After they share an idea they must justify their reasoning and explain their thinking. Then, other students can ask questions and analyze whether they agree with the other student's idea. The teacher's job is to encourage students to make conjectures about possible solutions or further steps. If the class is stuck, then the teacher will provide questions that may spur the thinking of students. The class is mostly in charge of solving the problem, but the teacher will provide some general guidance. This guidance involves facilitating the class's discussion of possible solutions. The teacher asks questions and provides slight prompts to help the students solve the problem by pointing out certain things they should be considering. These questions often include asking for clarifications to students' solutions and suggestions.

One of the main benefits to this stage is that it allows students to feel like they are a part of their own learning. They feel listened to and as if they are discovering the material. This stage is really good at engaging students. It allows them to develop questions based on their curiosity. Many students in math classrooms are discouraged from this kind of engagement in math class (Boaler, J., 2009). This stage allows students to learn based on their discovery and makes their input feel important and valued. This leads to them learning to explain their reasoning and deepen their understanding of the concept even further. Students are guided instead of strictly told what to do.

The collaborative learning stage is the next stage. This stage allows students to explore, gain clarification from peers, build discussion, and solve the problems given. Effective collaborative learning includes: positive interdependence, face-to-face interaction, individual and group accountability, interpersonals and small group skills, and group processing (Fisher, D. and Frey, N., 2014, pp. 69-70). Have students summarize, question, clarify, and predict (Fisher, D. and Frey N., 2014, pp. 82-83). For engagement you can: allow students to express ideas and thoughts creatively, allow students to develop strategies and uses of mathematical tools, and focus on process and explanation instead of just answers (Anderson, R., 2007). This engagement can be accomplished through encouragement and positive reinforcement. It is important to encourage each student to share their ideas and put emphasis on effort instead of accuracy when students discuss.

There are several benefits to the collaborative learning step. One benefit is that students can get explanations from their peers. This allows them to gain clarification in a phrasing that makes more sense to them. This also allows the student who is explaining to gain a better understanding of themselves. This is another chance for them to voice their thoughts and feel a part of the learning instead of just being lectured at. When students from a summer school were interviewed, many of the students claimed “that discussing ideas gave access to understanding and makes the discipline more interesting and engaging” (Boaler J., 2016, p. 4)

Independent learning is the final and simplest stage. It is the time set aside for students to practice on their own. This is a chance to have a safe environment for them to figure out their own level of understanding. This is also a chance to ask questions on the practice before class is over. Although students may not finish the entirety of the assignment before class is over, it is important to give them sufficient time to attempt or plan a strategy for each question before they leave. It is better to make sure problems are effective so fewer questions can be assigned. During this stage the teacher usually walks around and checks on the different students to see how they are all individually understanding the material.

Even though this is the final and simplest stage, independent learning is still very important. One of the benefits is that students are able to have time to develop their own questions and thought process during class, when they are still able to ask for help. Since this is the last step, they can use all of the ideas and concepts that have been discussed in the previous three stages to form their own process of solving the problem. Another benefit is that the students are able to practice different methods on their own and determine if they truly make sense to them. This also creates a safe time for students to ask for help when they don’t understand because it is not in front of the whole class. It also lessens the pressure on parents to help students with math homework since many parents may be busy and not able to help with the homework.

Although Gradual Release is a tremendous teaching structure, it is not fully effective on its own. Without the correct problems being used during the lesson and the right techniques to facilitate an effective discussion, many students will still struggle. It is important to provide problems for collaborative learning and independent learning that help students see math's relevance and encourage them to truly understand what they are learning. Also, in order for guided instruction and collaborative learning to be effective, the discussions need to be facilitated well. This helps students get engaged with the lesson better and feel safer to share their ideas.

Open Middle Problems

A type of problem that fits well with the Gradual Release structure is the Open Middle Problem. Open Middle Problems use the same mathematical concepts as other problems, but they are presented in more of a puzzle format. This promotes critical thinking, and encourages students to gain a deeper and fuller understanding of the material they are being taught. For example, an average problem in a book for Pre-algebra may be: 21+x=70, so solve for x. Instead, an Open Middle Problem with the same mathematical concepts would be something more like this: Using digits 1 to 9 at most one time each, place a digit in each blank to create two equations: one where x has a positive value, and one where x has a negative value _ _ +x=_ _ (Kaplinsky, R., 2020).

The big difference between the two problems is that the Open Middle Problem does not allow you to immediately compute using a formula. A student must look at the problem and truly understand what is happening. This means the student also can recognize the different ways the numbers used on each side relate to the numbers used on the other side. When a group of teachers, who taught a combined 1,120 students, were asked to present these problems to their students the results were startling. 92 percent of students were able to correctly find the answer for the first type of problem. However, when students were presented with the second problem, only 51 percent came up with a correct answer (Kaplinsky, R., 2020). That is a 41 percent drop in accuracy from a surface level problem to a problem that required a true and deeper understanding of the concepts involved. Then, the students were presented with a third problem which read: Using the digits 1 to 9 at most one time each, place a digit in each blank to create an equation where x has the greatest possible value. _ _+x=_ _. The accuracy of this problem was even lower than the second problem. Only 37 percent of students were able to find a correct solution (Kaplinsky, R., 2020). This not only shows the lack of true understanding by students, but also the ineffectiveness of the first problem to portray students' understanding accurately.

Another important difference between Open Middle Problems and problems traditionally used in the class is that they almost entirely eliminate randomly guessing. In many Open Middle Problems, randomly guessing would take way more time and effort than using reasoning. By eliminating the option to simply just guess and possibly get the answer correct, the student must think more deeply about the problem and what the teacher is trying to teach them. They must take steps to figure out how each attempt affected the result. Looking at this relation between attempt and result, students begin to see the patterns within math problems. By noticing these patterns, they start to see the deeper nature of the concept presented in the problem.

One of the most important features of Open Middle Problems is that they promote making mistakes. A student is given at least 3 or 4 attempts to solve a problem, and the final answer is only worth a small portion of the points for the question. The attempts are worth most of the points. This allows students to not put pressure on themselves to get the solution correct the first time. Not only does it allow students to make mistakes, but it teaches them how to respond to a mistake. For each attempt, the student is asked to explain what they learned from the attempt and what they will change for their next attempt. This teaches them to evaluate their mistakes and learn what to try differently the next time.

Open Middle Problems would work best for the Collaborative Learning or Independent Learning sections of Gradual Release. These problems would work well for Collaborative Learning because they involve critical thinking that could allow for several different approaches and thought processes. This allows for great discussion between peers on how they could solve the problem in several ways and make discussion about how their approaches relate to the learning objectives. It is also great for Independent Learning because it helps students and teachers evaluate how well the student understands the material as an individual. It gives a chance for the individual student to try different approaches as well. It also makes it so students can finish work in class most days, or it at least allows students to get a good start on homework before class ends. This is because it eliminates a large amount of homework because only a few Open Middle Problems are necessary. This helps students be able to have time to ask questions in class about their homework because they are having less problems to do.

<u>The 5 Practices for Orchestrating Productive Mathematical Discussions</u>

In addition to using effective problems such as Open Middle Problems while teaching with the Gradual Release method, it is important to use effective facilitation techniques to provide good classroom discussion during the Guided Instruction and Collaborative Learning phases. Five very important practices to improve the success of discussion are: anticipating, monitoring, selecting, sequencing, and connecting. These are considered the 5 Practices for Orchestrating Productive Mathematical Discussions (Smith, M. S., & Stein, M. K., 2018).

The anticipation part is all about predicting the different strategies and approaches your students will take towards the task you give them. This stage allows the teacher to see if the problem they have chosen for the activity will be fitting for the discussion activity by making sure there are several ways to solve the problem. When practicing the anticipation stage, the teacher must try to solve the problem in as many ways as they can (Smith, M. S., & Stein, M. K., 2018). This includes approaches that yield both correct and incorrect answers. It is important for the teacher to consider possible mistakes the students may make while trying to solve the problem. Because, by predicting mistakes, the teacher can prepare explanations to help with misunderstandings that students may have. Anticipation usually occurs before the class period takes place (Smith, M. S., & Stein, M. K., 2018).

Then, the monitoring aspect is when the teacher observes what their students are doing and how they are attempting the problem after it is assigned. This usually involves moving around the classroom and watching and listening to students as they work. This works well if students are either in small groups or individually. An equally important part of monitoring to listening and watching is asking students questions about their techniques and thinking. This not only benefits the teacher's understanding of their work, but it also helps the students develop their techniques more. These questions can be prepared beforehand based upon the anticipation stage. The approaches and the student thinking that occurs during the monitoring stage determines the following three stages (Smith, M. S., & Stein, M. K., 2018).

Next, during the selection stage, the teacher selects different examples that portray different mathematical concepts that they want to highlight in the lesson. This includes both mistakes that represent misunderstandings that need to be clarified, or works that represent learning objectives that your lesson wants to accomplish. The selection decisions are based upon what the teacher observes, listens to, and how students answered questions the teacher asked during the monitoring stage. Teachers are able to tell students or groups that they are selected before or during the discussion. Selection allows the teacher to have control and give the discussion structure, but it still allows students to feel ownership of the discussion (Smith, M. S., & Stein, M. K., 2018).

After that, the sequencing stage is where the best order to present each strategy that was used is determined. The order that is decided must allow the works to build upon one another. Place similar strategies closer to each other, so it is easier to see the connection between the strategies. Place strategies that are more common at the beginning to set a base level for the discussion, and present approaches that are more complex or less common later on. Then, it is easier to perform the connection stage later on between the complex and the more common techniques. It can also be helpful to more visual approaches first such as drawings, and then move onto more abstract approaches such as algebra. When these ideas are presented in an order to build off of previous ideas and make maximum connection to learning objectives, the final stage will be most successful (Smith, M. S., & Stein, M. K., 2018).

The final stage, connecting, is when the teacher connects the different strategies to each other, and you are showing the connections that the strategies have to your learning objectives. Along with making connections to learning objectives, it is crucial to make connections between different works by students. These connections are what will lead the discussion. This is all about students recognizing patterns between the approaches, different consequences or benefits of taking different approaches, and helping students figure out what approaches make the most sense to themselves. Consequences and benefits can include aspects such as accuracy, time required to carry out approach, or practicality. Some students may have different consequences and benefits with different strategies due to different styles of learning or thinking, so it is important to keep this in mind. Connection between different ideas and learning objectives helps students see patterns in mathematics and notice how free flowing and creative mathematics can be (Smith, M. S., & Stein, M. K., 2018).

Using the 5 Practices for Orchestrating Productive Mathematical Discussions has many benefits towards making a classroom more effective in teaching math. One of the main benefits is that it engages students. By showing students works, a teacher gives ownership of the discussion to the students. Students are more engaged when they feel like they have power and a voice in class, and by giving students a chance to express their thoughts or make clarifications about their work it gives them a reason to be more engaged in the class. Their understanding of the material is also going to increase because multiple approaches to solving a task are being presented. Students are also explaining the connections between different approaches and how they all connect to the learning objectives the students are supposed to learn. This gives students a chance to pick an approach that makes the most sense to them, and causes them to avoid simply memorizing because there is not one single strategy to memorize, there are multiple. Lastly, it helps students who have a fear of being embarrassed when asking a question because the teacher can ask for clarifications from students if none of the students ask. This way clarifications can be made without forcing a shy student to ask a question if they are not yet comfortable. It also shows more appreciation for mistakes when mistakes are presented to the class as a learning opportunity, and it gives students confidence that if they make a mistake it can still be beneficial to the class.

The 5 Practices are a great fit for the Gradual Release method. It works best during the Collaborative Learning section. This stage focuses on discussion, and using these practices helps provide the best structure for a successful and productive discussion. It can also be modified to be used for the Guided Instruction section as well. For use during the Guided Instruction section, instead of having students be in groups, the teacher can simply select conjectures, proposals for next steps, and other comments by students and connect them to each other and the learning objectives. In Guided Instruction, there would be more emphasis on anticipating possible strategies, selecting relevant comments and ideas, and both showing connections between ideas and prompting the students to see connections between ideas. For shorter classes that are around 50 minutes long, it is important to find problems that can be done within about a 10 minute period, so that there is enough time for discussion. There are also some concepts that don't always have multiple approaches, so it is important to understand that the 5 Practices may not fit all lessons. With these guidelines, the 5 Practices can help the Gradual Release method maximize the effectiveness of a math classroom.

Conclusion

It's been shown that many students today are struggling in math class. They are struggling to find the relevance of math or a reason to be engaged during class. Classrooms are not being taught to understand, but instead they are being taught how to memorize. Mistakes are being discouraged, and a large number of students are afraid to try because they don't want to risk suffering the consequences of getting an answer incorrect. By using the Gradual Release method, math educators will be able to create a better environment and provide students with the encouragement they need. Along with the Gradual Release method, using tools such as Open Middle Problems and the 5 Practices of Discussion are vital to meeting a math classroom's full potential. By using these methods teachers will be able make math classrooms more effective and better learning environments. Through time, these techniques can help take away the stigma and the fear around math, and allow students to view math as the creative and explorational subject that it is.

Appendix

Lesson #1

Common Core Standard: 8.F.A.1- Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

Focus Instruction (10 minutes):

- Procedure:
    - Write: learning objectives, key terms, relation to past learning, and intriguing questions on the board to the side before class.
    - Introduce the learning objectives, key terms, and explain both the connection to past learning and the concepts' purposes (3-4 minutes).
    - Walk through examples (3-4 minutes).
    - Refer back to the intriguing questions on the board about the example and allow a few students to give possible answers for those questions if time permits (2-4 minutes). If time does not permit, just tell students to keep those questions in their mind as the class goes on.
- Learning Objective(s):
    - Be able to provide 3 examples of how functions are used in the world
    - Be able to use a given data set to predict unseen data for a data set
- Key Terms: Function, input, output, and ordered pairs
- Explain purpose and use of concept:
    - Shows a clear relation between two things

- Helps represent real world data in a precise way
- Relation to past learning:
  - 7th Grade:
    - Ratios and Proportions
    - Expressions and Equations
- Intriguing Question(s):
  - What aspects of life may you be able to use a function for?
  - What is the relation between the input and the output?
- Example(s):
  - How long you work compared to your total income
  -

Hours Worked	1	2	3	4	5
Total Income in dollars	10.75	21.50	32.25	43	53.75

  - Some examples are not a consistent change such as how much it may cost to feed your pet if the price changes frequently
  -

Month	January (1)	February (2)	March (3)	April (4)	May (5)
Cost of pet food in dollars	30.75	29.50	30.50	29.99	30.75

Guided Instruction: (10 minutes)

- Procedure:

- Present following problem to class on board
- Ask class how they may solve the problem and give 1-2 minutes for class to think.
- Then allow students to discuss their proposals on how to solve and encourage other students to ask questions and think of other ideas as well.
- If students get stuck use prompts and ask guiding questions when needed.
- Allow as many ideas as time permits, but highlight ideas that connect well to previously shared ideas and the learning objectives.

- Example(s) to present to class:

Game	1	2	3	4	5	6
Total Points Scored Overall By Johnny	13	25	39	52	64	?

  - After game 6, about how many total points will Johnny have scored over the course of the 6 games?

- Prompt(s):
  - Look at the differences between individual outputs.
- Questions to promote thinking:
  - Which is the input and which is the output of this chart?

        - Do you see any patterns between inputs and outputs, between inputs and other inputs, or outputs and other outputs?
        - Before you estimate how many points Johnny scored after six games, what do you need to estimate first?
        - What may the next ordered pair be?
    - Anticipation:
        - Average change
        - Median in changes
        - Mode of changes
        - Trends of changes

Collaborative Learning (10-20 minutes):

- Activity:
    - Figure out the connection from input to output
    - Estimate the next data entry
    - Activity Layout:
        - Break students into group of 3-4
        - Present the following question on the board (or projector) and give students 10 minutes to come up with a solution and plan a brief 1-2 minute presentation of how they determined their solution.
        - Move around the room and monitor groups' work. Ask questions such as "why do you think that may work? " and "how do you

plan to use that strategy with this particular problem?" If everyone finishes early, move on to presentations.

- Choose 3-5 groups to present to the class their method of solving the problem. Select groups who have done different strategies such as finding the average, mode, median, trends, or a method unexpected.
- Then have groups present in the order of complexity (generally in the order of mode, median, trend, average, and unexpected method). If you select a group who made a mistake (which is preferred if possible, have them present earlier on and use the mistake as a learning opportunity for the class and thank the group for their work).
- As groups present take 1-2 minutes in between presentations to allow students to make comments, explain connections, or ask for clarifications. Allow students to answer these questions if possible.
- Grading System: 10 points for participation

- Questions:

The following table shows the amount of money Juan finds in his clothes pockets while doing laundry. Estimate how much Juan will have on Week #10.

Week #	1	2	3	4	5	6	7	8	9	10
Money found	$1.27	$0.50	$0.10	$0.50	$0.50	$0.20	$0.27	$0.00	$0.05	?

Anticipate: Possible ways to estimate how much money was found on week #10.

1. Find the average money found and use that as an estimate.
2. Guess that it will be the amount of change that has been found the most (mode).
3. Notice an overall trend of how the money is slowly getting less.
4. Find the median.

Independent Learning (10 minutes or however much time is left):

- Procedure:
    - Present the following questions on a small handout and have students turn it in when done.
    - Move around the room to check-in on students to see if they are struggling or not.
    - Grading System: 7 points for completely attempting per question and 3 points per question for accuracy (total of 10 points per question).
- Question: What are three examples of what functions could be used to present in the world outside of the examples already provided? (You may not use: income in relation to hours worked, cost of pet food, points scored in basketball in relation to games played, or money found in laundry in relation to which laundry load it is).
- Questions: Create a table with a consistent increase or decrease so that for the last input, the output is 60.

Example:

Input	1	2	3	4

Output	15	30	45	60

Lesson #1 Explanation

For my first lesson plan I had reasons for all of my choices for the focused instruction section. First, my objectives were chosen because they were measurable and specific. This is important because if a student is able to complete them it shows they have a better understanding of the concepts. It also helps students see how the objectives relate and connect to other concepts (Smith, M. S., & Stein, M. K., 2018). The objectives I chose also helps students connect the learning to real life examples. If a student is able to think of real life examples, it helps them see the connection between math and the real world, and it helps students use critical thinking. Having another objective about being able to predict is also good because prediction uses critical thinking when it is not a clear formula where you just plug in a value to get the answer. Secondly, I chose the particular key terms: function, input, output, and ordered pairs for this lesson because those terms are the foundation for understanding the concepts of functions. The terms input, output, and ordered pairs are important for students to understand before they learn how functions can look as equations and graphs. These are also terms that can easily be referenced to the rest of the mini unit. Next, the intriguing questions I chose were chosen to both help students focus on learning the relationship between the input and output and think about how the idea of functions works in the world. These questions allow students to relate the math to topics they care about and give them motivation to learn. It also helps guide them in knowing what is the most important takeaway from the lesson.

Lastly, for the focused instruction, I had to select an example that would be most beneficial for students' foundational understanding. It also had to leave opportunity for students to continue exploring and feel curiosity about the subject. The examples I chose didn't have any solving that had to be done. I simply just showed what a couple examples of inputs and outputs in the real world may be. The first example was a constant rate of change where you compared your earnings with the hours you've worked, and the second was a non-constant rate where you compare pet food prices with the month of the year. These help students see examples of inputs and outputs without taking away their ability to think on their own later in the lesson when problem solving comes into play. It is also beneficial for students to see from the beginning that not all data sets are constant rates of change.

My guided instruction section also was thought out. I chose a problem for the class to work through where students had to predict the total amount of points scored after the sixth game of basketball. I chose this problem because there is no right answer. Students can predict how many points will be scored in the sixth game from a variety of strategies which helps develop good discussion and provides opportunity for students to practice justifying their reasoning. The prompt I chose was to tell students to focus on how the output differs from game to game. I chose this to help students think about how to use the difference of outputs from previous games to think about how it may help them predict the difference between game 5 and game 6. Finally, for the guided instruction, I had to choose the questions to promote thinking. The questions I chose had to do with guiding students in thinking about what steps to take first and help them think about aspects of the problem that were most important. I chose the questions I do in order to guide students, but still leave a chance for them to fill in the gaps of what steps to do and why those steps are beneficial. My questions also help students connect back to the definitions for the day which helps them learn the vocabulary better. In addition, asking guiding questions can help the teacher identify what the class as a whole is understanding or misunderstanding (Fisher, D. and Frey N., 2014, pp. 41).

For the collaborative learning phase I chose a table showing the amount of change Juan finds in his laundry each week. The box for the tenth week is left blank for the students to estimate how much Juan will find on the tenth week. I chose this question because it is very open ended. There are several different ways that students could choose to estimate the change. It creates a safe atmosphere where no answer is incorrect. This puts more emphasis on explanation and process instead of the answer. Putting emphasis on explanation allows for more students to share their ideas and be engaged (Anderson, R., 2007). Also, since there are several different ways to answer the question, it makes for a great conversation question for students to present their groups' strategies to the class. It is also a low floor and high ceiling problem because it doesn't take advanced math skills to estimate. However, advanced math skills can be applied still if a student or group wishes too. This problem is also a great connection to past learning because groups can use mean, median, or mode which they've learned in the past most likely. It is also a very simplified example of what an input and output may be in real life, and it shows that not all inputs and outputs have a clear pattern. This takes away rigid use of formulas and promotes critical thinking and open discussion.

Lastly, for the independent learning phase I had two problems for students to solve. The first problem asked students to describe three examples of functions in real life that weren't used in class. I chose this because it gave students an opportunity to relate the learning to their own lives. It also gives students a chance to think deeper in the material. Most students could regurgitate examples from the lesson, but to come up with new examples takes more thought. The second problem asked students to create a table with a consistent increase or decrease, so that the last input has an output of 60. I chose this because there many different possible answers and it gives a very subtle informal introduction to slope. Even though slope isn't mentioned directly, this is a chance for students to practice working with rates of change in a simplified version. This gives an opportunity for later for students to refer back to this problem when later learning about slope.

Lesson #2

Common Core Standard: 8.F.A.1- Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

Focus Instruction (7-10 minutes):

- Procedure:

  - Write: learning objectives, key terms, relation to past learning, and intriguing questions on the board to the side before class.
  - Introduce the learning objectives, key terms, and explain both the connection to past learning and the concepts' purposes (2-4 minutes).
  - Walk through examples (3-4 minutes).
  - Refer back to the intriguing questions on the board about the example and allow a few students to give possible answers for those questions if time permits (2-4 minutes). If time does not permit, just tell students to keep those answers in their mind as the class goes on.
- Objective(s):
  - Graph points on a graph from a data set
  - Be able to determine simple equations based on inputs and outputs from a table
- Key Terms: Ordered Pairs, output (y), input (x)
- Explain purpose and use of concept:
  - Present data in a new way
  - Present data in a visual way that shows relation
- Relation to past learning
  - Last lesson with inputs and outputs
  - 7th Grade:
    - Ratios and Proportions
    - Expressions and Equations

- Lesson #1:
  - How inputs and outputs relate
- Intriguing Questions: How do, tables, equations, graphs, and ordered pairs relate to each other?
- Possible errors to be aware of:
  - Make sure you remember the input (x) is the first in an ordered pair
- Example(s):
  - Graph the following equation for x values 1-4: y=3x
    - (1,3),(2,6),(3,9),(4,12)
    - While graphing, explain that x values are set for the horizontal line and y values are set for the vertical line.

Guided Instruction (10 minutes):

- Procedure:
  - Present following problem to class on board
  - Ask class how they may solve the problem and give 1-2 minutes for class to think.
  - Then allow students to discuss their proposals on how to solve and encourage other students to ask questions and think of other ideas as well.
  - If students get stuck use prompts and ask guiding questions when needed.

- Allow as many ideas as time permits, but highlight ideas that connect well to previously shared ideas and the learning objectives.

- Example(s):Write out this data set as a list of ordered pairs, determine an equation (or formula) for the function, and then graph the ordered pairs
  -

Number of eggs	2	4	6	8	10
Total Cost	1	2	3	4	5

  - (2,1),(4,2),(6,3),(8,4),(10,5)
- Prompt(s):
  - Think about which numbers are inputs and which are outputs.
- Questions to promote thinking:
  - What is happening to the number of eggs in relation to the total cost?
- Possible Struggle Points: Figuring out which is the output and input
- Brief Explanations: The input is what causes the output. Since the number of eggs affects the total cost, the number of eggs is the input.

Collaborative Learning (10-20 minute):

- Activity Layout:
  - Break students into group of 3-4
  - Present the problems listed below on the board (or projector) and give students 10 minutes to come up with a solution for each part.

  - While they work, walk around the classroom and ask groups to make sure that their data set, equation, and graph all match. Select 3-4 groups to present one of their graphs. Try to pick 1-2 groups who have made a mistake with one of their graphs. Pick 1-2 groups to present their solution for Problem #1 and 1-2 to present their solution to Problem #2.
  - Presentations should include: showing/telling the class what their data set, equation, and graph were and explaining their thought process for coming up with a solution.
  - Then, allow about 1-2 minutes for each of the selected groups to share their data set, equation, and graph for the problem they were selected to present with the rest of the class.
  - Have groups presenting Problem #1 solutions go first, then Problem #2 solutions. Try to have students with errors present their work before their peers for their respective problem. Connect errors to other solutions to help clarify misunderstandings.
  - Grading system: 10 points for participation
- Problem #1: Create a data set with 5 inputs and 5 outputs that when graphed is a diagonal line that goes down the further right you go. Determine what the equation of the line is.
- Problem #2: Create a data set with 5 inputs and 5 outputs that when graphed is a flat horizontal line. Determine what the equation of the line is.

Independent Learning (10 minutes or however much time is left):

- Procedure:
    - Present the following questions on a small handout and have students turn it in when done or the next class period. Have students write out work on a separate piece of paper. Students must apply at least 3 attempts to get full credit for each question (more than 3 attempts is okay). After each attempt students must explain what they learned from that attempt and what they will try differently for the next attempt. After their final attempt they must say why they think their answer is correct. Before they leave make sure they get at least one attempt per question, so they can ask questions to get clarification before they leave.
    - Move around the room to check-in on students to see if they are struggling or not.
    - Grading system: 2 points per attempt, 2 points per explanation after each attempt, 2 points for accuracy for a total of 14 possible points per problem.
- Problem: You are given the equation $y=1/x^2$ and the inputs: $x_1=\_$, $x_2=\_$, $x_3=\_$. You may use the digits -9 to 9 and you may only use one digit per blank space. Your goal is to fill in the blanks so that the y value of x_2 is less than the y value of x_1, and that the y value of x_3 is greater than the y value of x_2.

Answer: $x_1<0$, $x_1< x_2<x_3$, and $x_3>0$.

Lesson #2 Explanation

For my second lesson plan I used the same format as the first lesson plan. For the focused instruction's learning objectives, I only went with two in order to focus the learning of the lesson. These objectives of "graph points on a graph from a data set" and "be able to determine simple equations based on inputs and outputs from a table" were chosen because they are specific acts that can be measured through some kind of assessment, in this activity informal. These also build off of the previous objectives which introduce inputs and outputs and the idea of functions. This lesson's objectives now introduce the concept of how to use inputs and outputs for a graph. For the key terms for lesson #2, I used three of the same words from the previous lesson. This helps keep this vocabulary in the students' minds. This lesson also builds on the ideas of ordered pairs, inputs, and outputs from last lesson. It still uses the same concepts from the last lesson where the inputs and outputs are seen in tables. This allows a chance for students who missed class the previous day to get caught up much more easily, since the previous lesson's use of the terms is still being used. I also added (x) and (y) by input and output in order to make the connection for students that inputs are on the x value and outputs are the y value. This gives their heads a head start for when we start graphing points.

For the focused instruction, I also had to choose the intriguing questions. For my second lesson plan, I only used one intriguing question for the focused instruction. This question asked about the relation between tables, equations, and graphs. I decided to ask this question in order to get students to start thinking about upcoming material.

This gives them a chance to be curious for what they are about to learn and be introduced to later in the lesson. This gives students a chance to make connections before the class and group discussions occur in guided instruction and collaborative learning. By asking this question, students can use their own thinking to imagine possible relations between tables, equations, and graphs before they are explained. If students are simply told the relation, they will be less engaged and will lose their motivation of curiosity. By waiting, it provides them with motivation to continue engaging in the lesson to see if their predictions are correct.

Next, the problem I chose for the focused instruction was graphing the first four points of the equation y=3x. This gives the students a basic introduction of how inputs and outputs work with equations and how an equation can get graphed. I chose this problem because it gives a brief introduction for students on what they will be doing for the day. There is a large amount of content in this problem, but since students have three phases left of class to gradually work on tackling this content more and more on their own, this is a good introduction for them to get their brain thinking. Although the relations are explained in this example, there will most likely be a lot of questions. Because of the amount of questions that will likely be asked, the guided instruction, collaborative learning, and independent study phases allow for a great opportunity for students to wrestle with the material and slowly begin making connections to the focused instruction example. The focused instruction is not meant to provide total learning, it is just to give foundational understanding and introduce content.

I also had to choose a problem for the guided instruction. For this problem, there is a table that shows the number of eggs in relation to the total cost. Students are asked to write out the ordered pairs and graph them. I chose this problem because it is a basic example of inputs and outputs in a table that can be set into ordered pairs. This is also a good introduction to how tables can be formed into graphs, and it shows the relation between the table and graph forms of a function. The prompt for this problem tells students to focus on determining which numbers are inputs and outputs. This helps students make sense of what an input means and why it helps create an output. The guiding question asks what the relation is between the eggs and the total cost is. This informally introduces the idea of slope, but total understanding of this question is not mandatory to accomplish the task. The teacher is able to see how well students understand the idea of slope before it's currently taught. It also helps to see if students understand or misunderstand the important aspects of the lesson (Fisher, D. and Frey N., 2014, pp. 41). These questions also help create something for students to reference in future lessons when slope is formally defined and explained in more detail.

The collaborative learning phase of this lesson had two different problems for groups to work on. The problems asked students to create a data set with 5 inputs and 5 outputs that first create a diagonal line when graphed and then create a straight horizontal line when graphed. After graphing each set of data points, the groups are asked to create an equation for each data set. I chose these problems because they are a good introduction to y-intercept equations without formally explaining them to the class. Students can figure out an equation based on the patterns they see, and they don't need to already know about y-intercept form equations yet. It is also a very good and basic example of how tables, equations, and graphs relate to each other. It also helps students visualize what graphs imply when they are going down or are horizontal lines. A declining graph implies a decrease and a horizontal graph implies that the data is staying the same.

For the independent learning section I provided an Open Middle problem for students to complete. This problem asked students to pick three different x values for an equation such that the y values had certain relations to each other. I chose this problem because it helps students see how inputs can create different types of outputs depending on the function. In order to solve this problem, students must fully understand what happens in the function when inputs get larger or smaller. Students are also required to attempt the problem several times and to write explanations after each attempt on why it did or didn't work. This puts emphasis on making mistakes and sends a positive message about making mistakes. Giving a positive message about making mistakes can greatly benefit a student's learning in math (Boaler, J., 2016, p. 17). It is also important for students to write out their thoughts. This way, they can reflect on what does or doesn't work for the problem. It helps them practice looking at math as a process of learning and not something they must understand immediately.

Lesson #3

Common Core Standards: 8.F.A.3- Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

8.F.A.2- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

Focused Instruction (10 minutes):

- Procedure:
    - Write: learning objectives, key terms, relation to past learning, and intriguing questions on the board to the side before class.
    - Introduce the learning objectives, key terms (don't explain y-intercept or slope yet), and explain both the connection to past learning and the concepts' purposes (2-3 minutes).
    - Walk through Desmos examples and intriguing questions about m and b (3-5 minutes). After students get a chance to answer, explain slope and y-intercept.
    - Refer to the last intriguing question (How does what you just saw relate to tables?) on the board and allow a few students to give possible answers for those questions if time permits (2 minutes). If time does not

permit, just tell students to keep those answers in their mind as the class goes on.

- Objective(s):
  - Be able to graph a linear equation
- Key Terms: Slope,y-intercept, linear equation
- Explain purpose and use of concept:
  - To express a set of data in a new way
- Relation to past learning:
  - Lesson # 1 and #2
    - What a function in the shape of a table looks like
    - Inputs and outputs
    - Graph, equations, and table connections
- Intriguing Activity/Question(s):
  - Go on Desmos and plug in different m and b values for an equation.
  - What happens when I change m?
  - What happens when I change b?
  - What do you think m and b represent?
  - How does what you just saw relate to tables?

Guided Instruction (10 minutes):

- Procedure:
  - Present following problem to class on board

  - Ask class how they may solve the problem and give 1-2 minutes for class to think.
  - Then allow students to discuss their proposals on how to solve and encourage other students to ask questions and think of other ideas as well.
  - If students get stuck use prompts and ask guiding questions when needed.
  - Allow as many ideas as time permits, but highlight ideas that connect well to previously shared ideas and the learning objectives.
- Example:
- Create both an equation and a graph for the following table of data:
-

Cookies Eaten	1	2	3	4	5
Total Calories	60	120	180	240	300

- Prompt(s):
  - You can’t see the y-intercept on the table, but think about the logical answer for how many calories you’d intake after eating zero cookies.
  - Look at the change in calorie intake compared to cookies eaten.
- Questions to promote thinking:
  - How do you scale out the graph to make these higher numbers fit?
  - What is the slope of this data, and what does the slope represent?

  - What does the y-intercept represent for this data and what is it?
- Possible Struggle Points:
  - May struggle with finding y-intercept since 0 is not on table.
  - May not be sure how to graph such large numbers.

Collaborative Learning (10-20 minutes):

- Break students into group of 3-4
- Present the following question on the board (or projector) and give students 10 minutes to come up with a solution and plan a brief 1-2 minute presentation of how they determined their solution.
- Move around the room and monitor groups' work. Ask questions such as "why do you think that may work? " and "how do you plan to use that strategy with this particular problem?" If everyone finishes early, move on to presentations.
- Choose 3-5 groups to present to the class their method of solving the problem. Select groups who have done different strategies such as creating a table, equation, graph, or a method that was unexpected.
- Then have groups present in the order of complexity (generally in the order of table, equation, graph, and unexpected method). If you select a group who made a mistake (which is preferred if possible), have them present earlier on and use the mistake as a learning opportunity for the class and thank the group for their work. Likely mistakes are using the trend that in the first four Pattern #1 is always greater than Pattern #2 and assuming that it will be the same case for Phase #10. If this mistake comes up, have that group present first. They may

also make their equation based upon just one or two phases, and if that occurs have that group present before the other equation group.

- As groups present take 1-2 minutes in between presentations to allow students to make comments, explain connections, or ask for clarifications. Allow students to answer these questions if possible.
- Grading System: 10 points for participation
- Questions:
- Look at the following patterns of squares. You are given the first four phases of each pattern. Given what you know about functions and the different ways to represent them, which pattern will produce the most squares on the 10^{th} phase?

  Pattern 1: 10,20,30,40,...

  Pattern 2: 2,6,12,20,...

  Anticipation:

  1. Fill out table with all ten phases for each pattern
  2. Create an equation that matches the patterns
  3. Graph the points and graph equation

Possible errors:

1. Predict that since pattern 1 is greater than pattern 2 for the first four phases, pattern 1 will be greater at the 10th phase as well.
2. Make an incorrect equation based on only one or two inputs and outputs instead of all four collectively.

Independent Learning (10 minutes or however much time is left):

- Procedure:
  - Present the following questions on a small handout and have students turn it in when done or the next class period. Have students write out work on a separate piece of paper. Students must apply at least 3 attempts to get full credit for each question (more than 3 attempts is okay). After each attempt students must explain what they learned from that attempt and what they will try differently for the next attempt. After their final attempt they must say why they think their answer is correct. Before they leave make sure they get at least one attempt per question, so they can ask questions to get clarification before they leave.
  - Move around the room to check-in on students to see if they are struggling or not.
  - Grading system: 2 points per attempt, 2 points per explanation after each attempt, 2 points for accuracy for a total of 14 possible points per problem.

Questions:

Problem #1: Using the digits between -9 through 9 (excluding 0) determine two ordered pairs, point A and point B, that when connected by a linear line, creates a line with the equation (y=mx+b) that has a negative y-intercept value (b). You may only use each digit once per attempt. Point A (_,_) Point B (_,_).

Problem #2: Using the digits between -9 through 9 (excluding 0) determine two ordered pairs, point A and point B, that when connected by a linear line, creates a line with the equation (y=mx+b) that has a negative y-intercept value (b) and a slope (m) that is a whole number. You may only use each digit once per attempt. Point A (_,_) Point B (_,_).

Lesson #3 Explanation

For the focused instruction of my third lesson plan, I went through the same process as the last two lessons. First, my learning objective was to "be able to graph a linear equation". I chose this objective because it is a large objective that needs at least a whole lesson to focus on. This objective also can help fill in the gaps of anything still left misunderstood from the previous lesson. It is a good objective that helps relate to past and future objectives and concepts which is very important (Smith, M. S., & Stein, M. K., 2018). Most linear equations are similar to the example in lesson #2 where I taught them how to graph points for the equation y=3x. The only major difference is that the y-intercept is not 0 in the examples that are used in lesson #3. Therefore, this lesson can students understand the previous lesson and move forward into expanding the previous concepts from general equations to specifically linear equations. This objective is easy to connect to the learning done in the previous lesson by explaining how the equations explored in the last class are linear equations. The key terms are: slope,y-intercept, and linear equation. I chose these terms because they are the most important vocabulary for understanding the formation of a linear function. These key terms also give more meaning to the equations that were explored the day before.

They also are very easily displayed visually using a graphing software like desmos. As the slope and y-intercept change, students can pick up on trends.

The activity I chose for the focused instruction was to use desmos to display some linear graphs. I would write out the equation of y=mx+b on the board and on desmos. Then, I would fill in different values for m and b. I chose this activity in order to give students a visual view of what happens when m and b are changed. This gives them a chance to first see trends and then make predictions about what m and b could possibly represent. This raises their curiosity and allows them to collect their own thoughts before being told the answer. My intriguing questions are very tightly connected to the activity. They include asking about what happens when m and b are changed and what m and b can possibly represent. These questions guide students to think about the aspects of the activity I want them to form predictions on and indicate what aspects are most important to focus on during the lesson. I also asked how the activity relates to tables in order for students to think about how to connect the learning to past and future learning. This gives students a chance to build their thoughts before entering the next activity in guided instruction that connect tables and linear equations.

For the guided instruction problem, I decided to give a table that showed the number of cookies eaten in relation to the number of calories and asked students to create an equation and a graph that represents the table. I chose this problem because it is a linear example of a function. I also chose it because the class doesn't need a full understanding of slope and y-intercepts to be able to complete the task. Students can simply notice that the calories is the number of cookies eaten multiplied by 60. For graphing the table, students can plot points on the graph too. However, students who are a little further ahead can notice the slope and y-intercepts in these examples already even though full understanding of these terms isn't required to be able to complete the task.

For the prompts and guiding questions, I chose to have two prompts and three guiding questions. For the prompts I chose "you can't see the y-intercept on the table, but think about the logical answer for how many calories you'd intake after eating zero cookies" and "look at the change in calorie intake compared to cookies eaten". I chose the first one because some students may be focusing too much on the y-intercept. The first prompt helps them come to the conclusion that they don't need to worry about it because it is zero. The second prompt is meant to encourage students to look at changes in outputs in relation to changes in inputs in order to recognize the rate of change. For one of the guiding questions, it asks how to scale the graph to make the data fit. I chose this question because some students may struggle with the idea of using marks to represent intervals greater than one. The other questions asked what the slope and y-intercepts were and what they represented in the problems. This helps students think about what the numbers actually mean. This prevents students from blindly doing calculations and thinking more logically about what the calculations express.

The collaborative learning phase provides 2 patterns. Each group's responsibility is to determine which pattern will have the greatest value in the tenth phase. I chose this problem because it has multiple ways to determine the correct answer. Because of this it gives groups the opportunity to use any of the forms of functions that they have learned in the past few lessons. This gives students a choice of using whichever technique works best for students in their group and provides opportunities for individual thinking. Also, since there are multiple strategies that can be used this problem is great for conversation and side by side comparison between the different strategies. During a discussion the teacher can describe how the different function forms that can be used in strategies relate to each other and how they all show the same information.

The independent learning section contains two Open Middle problems that ask students to fill in the blanks of two ordered pairs. For the first problem, the two ordered pairs must make a line with a negative y-intercept, and the second problem asks for two ordered pairs that have a negative y-intercept and a slope that is a whole number. These problems are good because they build off of each other, but the second problem can still be done without figuring out the first problem. These problems help students practice finding the slope of a line connecting two points, and it helps students see how points of a line affect slopes and y-intercepts. These problems are good because they allow students to explore different possibilities while trying to solve the problem. These two problems have multiple correct answers, so there is freedom for students to use their own thinking and pathways. Writing out their thoughts after each attempt helps them understand why certain points don't work and make connections on why other points will possibly work. Instead of just trying to solve a problem, students are able to continue learning while doing practice problems. Explanations are also good because students can get correct answers sometimes without fully understanding. When students give explanations it helps show how much they truly understand. (Kaplinsky, 2019).

Lesson #4

Common Core Standard: 8.F.A.3- Interpret the equation *y = mx + b* as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

8.F.A.2- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

Focused Instruction (10 minutes):

- Write: learning objectives, relation to past learning, and intriguing questions on the board to the side before class.
- Introduce the learning objectives, and explain both the connection to past learning and the concepts' purposes (1-2 minutes).
- Walk through madlib and fill it in with class (3 minutes).

- Based on the written word scenario ask the intriguing questions and allow some responses from students as long as time permits (5ish minutes)
- Objective(s):
    - Be able to create word descriptions to match tables, equations, and graphs and visa versa.
- Key Terms: input (x), output (y), y-intercept, and slope
- Explain purpose and use of concept:
    - This allows you to use equations and graphs to easily represent real life situations visually
    - Also helps you verbally explain what is going on in a graph
- Relation to past learning:
    - Inputs and outputs
    - Relation of tables, equations, and graphs with functions
    - Use of y=mx+b
- Intriguing Questions:
    - What might the y-intercept be in this written scenario?
    - What could be a negative y-intercept in a word problem?
    - What might the slope (m) be in this written scenario?
    - What might a negative slope imply?
- Example(s): They create their own word scenario of a function in a madlib style way.

- Madlib: _______(Person) started out with $_____(number). Then they went to ______(Place where money is spent) and bought/sold (choose one) ____(number) _________(item(s)). Each _____(Previous item) cost $_____(number).

Guided Instruction (10-15 minutes):

- Present the following problem to class on board
- Ask class how they may solve the problem and give 1-2 minutes for class to think.
- Then allow students to discuss their proposals on how to solve and encourage other students to ask questions and think of other ideas as well.
- If students get stuck use prompts and ask guiding questions when needed.
- Allow as many ideas as time permits, but highlight ideas that connect well to previously shared ideas and the learning objectives.
- Example: Use two different methods to solve the following word problem:
    - Jessica really enjoys having cats. She currently has two cats and it cost her $22.00 a month to feed them. She has just moved into a new place that allows enough space for five cats now. If she only has $50.00 to spend for food each month, will she be able to afford to feed five cats? If she can't afford five cats, how many can she afford?
- Prompt(s):
    - Try to figure out the cost of a single cat per month.
    - Make sure to remember that she can't own a fraction of a cat.

- Questions to promote thinking:
    - What forms of functions have we used so far?
    - How can you find out how much each cat costs to feed?
- Possible Struggle Points:
    - Coming up with a second way
    - Finding cost of a single cat
    - Remember they must round down
- Anticipated ways:
    - Graph
    - Table
    - Equation and plug in 5
    - Other form of algebra

Collaborative Learning (10-20 minutes):

- Activity Procedure:
    - Put students into groups of 3-4.
    - Present the instructions listed below on the board. Allow about 5-10 minutes for students to make their word scenarios, graphs, and equations.
    - Monitor each group to observe how well their word problems, graphs, and equations are matching each other.
    - Select 3-4 groups solutions to be presented. If possible choose 1-2 solutions that don't match and have those be presented first. Then

choose another 1-2 solutions that do match and have those presented after. Allow about 1-2 minutes per presentation, so there are a few minutes to make comments about the variety of functions, ways to improve solutions to make them match better, or any other comments or questions.

- Activity:
    - Groups create their own equation
    - Next each group creates a word scenario to represent their equation
    - Next student creates graph for the word problem/equation

Independent Learning (5-10 minutes or however much time is left):

- Procedure:
    - Present the following questions on a small handout and have students turn it in when done or the next class period. Have students write out work on a separate piece of paper. Students must apply at least 3 attempts to get full credit for the first question (more than 3 attempts is okay), and they must apply at least 2 attempts for the second question for full credit (more than 2 is also okay). After each attempt students must explain what they learned from that attempt and what they will try differently for the next attempt. After their final attempt they must say why they think their answer is correct. Before they leave make sure they

get at least one attempt for the first question and review the second question, so they can ask questions to get clarification before they leave.

- Move around the room to check-in on students to see if they are struggling or not.
- Grading system: 2 points per attempt, 2 points per explanation after each attempt, 2 points for accuracy for a total of 14 possible points for the first problem. The second problem is 2 points per attempt, 2 points per explanation after each attempt, and 2 points for accuracy for a total of 10 possible points for the second problem.

Question 1: Fill in the blanks to the following scenario. Abed starts out with $_ and each day for _ days his parents give him $_. Fill in each blank with one digit between -9 to 9 so that if you write this scenario as a linear equation it would have the largest possible slope and Abed would end with the greatest possible amount of money. You may only use a digit once per attempt.

Answer: Abed starts out with $7 and each day for 8 days his parents give him $9.

Question 2: After you have figured out the first question, fill in the blanks of this scenario, so that if you were to graph both scenarios, they would be parallel: Troy starts out with $_ and he mows _ lawns for $_ for each lawn mowed .

Lesson #4 Explanation

For my focused instruction for lesson #4 I had to decide on the objectives and key terms. The objective I chose was “be able to create word descriptions to match tables, equations, and graphs and visa versa”. I chose this because it helps combine all of the learning from the previous three lessons and helps relate it to word problems. Relating the different forms of functions to word descriptions help students practice verbalizing what functions mean in the real world. Being able to describe a function with words shows a true understanding of what the table, equation, or graph is showing about a particular pair of inputs and outputs. This learning objective gives a chance for students to evaluate how deeply they understand the past three lessons' content. For the key terms, I chose not to introduce any new terms because there are no new terms to explore. With word descriptions, the terms they know are merely taking a different form than the past lessons. As a result, I kept the key terms as: input (x), output (y), y-intercept, and slope. This gives students a reference to the past few lessons and helps students understand them on a deeper level by seeing written descriptions or trying to describe the functions on their own.

For the activity of the focused instruction phase, I chose to have the class fill in the blanks of a word problem in a madlib format. This gives students a chance to participate and have fun with the activity. The students can be goofy and include random objects and prices. It also gives students a view of how functions can represent anything their imagination can think of. The intriguing questions guide students to think about what the slope may be in their made up scenario, what the y-intercept, input, or outputs may be as well. This connects the learning to the students' imagination and gives them a chance to think deeply about what the different aspects of a function would be for their scenario. I also planned a question asking about what a negative slope may imply. The idea of a negative slope is something students will be able to wrestle with, and is very important in helping students make sense of their answers later on when defining slopes of functions. This is important because oftentimes negative slopes can be confusing and this question can help students think about what times a negative slope would be an appropriate answer and what times it wouldn't be for future problems.

For the problem for the guided practice section of the lesson plan I chose a problem about how many cat's a person could feed with a given budget. The students are given the cost to feed two cats is $22 and are told that the budget is $50. The problem also must be solved in two different ways. These ways will most likely include either tables, equations, graphs, or some other form of algebra. I chose this problem because students have to think critically about this problem. They have to figure out what steps to take and how to complete those steps. First they have to figure out the cost of a single cat and then figure out how many cats can be fed with the given budget. It is also a good example of a slope where the $11 per cat is the slope. I chose to have the class use two different ways to solve the problem because it helps them explore the similarities that the different forms of functions have with each other. This also helps get more students involved. If the class only has to find one way to solve a problem, then the process ends quickly and the conversation gets limited. If there are multiple ways then it allows other ideas to be implemented into the strategy more easily.

Along with this problem, I also chose prompts and guiding questions to help the class if they get stuck. For the prompts I chose to mention that students should figure out the cost of a single cat to feed and to remember that there can't be a fraction of a cat. I chose these prompts because they help guide students on their first step to solving the problem and remember to use logic in their problem solving. The questions I chose to ask were “what forms of functions have we used so far?” and “how can you find out how much each cat costs to feed?” These questions help students reflect on past learning and use the past lessons to help them think of possible ways to solve the problem at hand. The second question helps students consider they need to solve first to move forward with the problem. The questions and prompts for this problem help students use logic and reflect on past lessons to figure out which first steps to take.

For the collaborative learning phase of the lesson groups are asked to create their own equation. From their equation, they need to create a word scenario that the equation could represent and graph the equation. This activity is good for this section of the lesson for several reasons. The first reason is that there are unlimited answers that students can come up with. This takes away the mindless step by step work that many students do in math classrooms. Instead, students get to use their own creativity and create something on their own. This gives students ownership of their learning. Groups also have to think about what types of equations they know how to do based on past learning, and it helps them reflect on what they know. Students also have a chance to get curious about how to put equations such as exponential equations into word problems or onto graphs. The next reason is that it gives good practice for students to use different function forms to represent the same data. Finally, it gives students a chance to have good discussion and share their own thinking and listen to both the thinking of group mates and other groups.

I had two Open Middle problems for the independent learning section. These both were filled in the blank word scenarios that represented functions. They asked to fill in the blank to get desired slopes and y-intercepts. These problems were good practice for students identifying the slope and y-intercept of a word scenario, and it also helped students practice translating between different forms of functions. The problem helps students understand on a deeper level how changing the slope or y-intercept of a function can drastically change a situation that a function is representing. The multiple attempts also encourage students to make mistakes and be creative in their thinking. It is also very important how students are required to explain their process after each attempt. This helps them think deeper into what went wrong and what they could try next time that could be more effective.

Lesson #5

Common Core Standard: 8.F.A.2- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

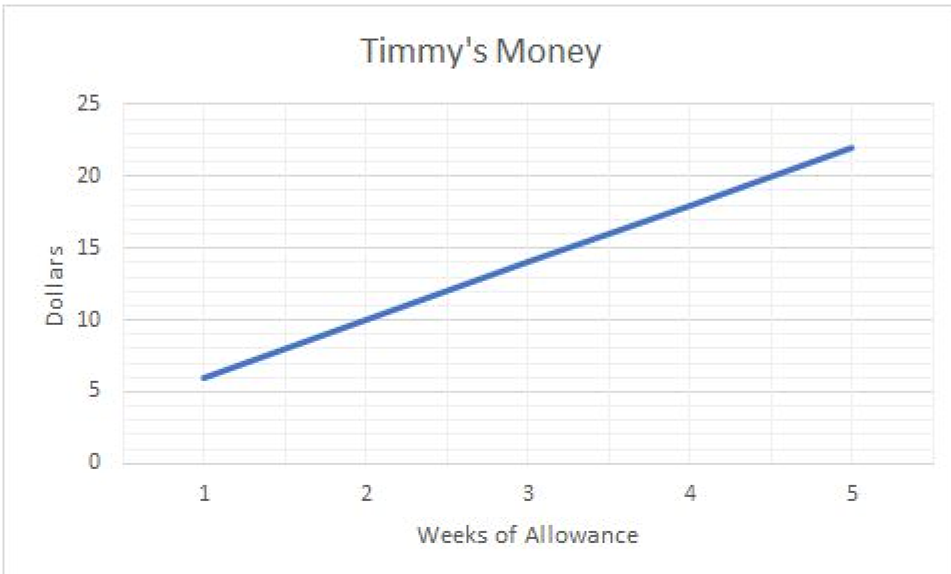
Focus Instruction (10 minutes):

- Procedure:
    - Write: learning objectives, relation to past learning, and intriguing questions on the board to the side before class.

  - Introduce the learning objectives, and explain both the connection to past learning and the concepts' purposes (2 minutes).
  - Walk through an example of how to find the slope of a linear graph (4-6 minutes).
  - Refer back to the intriguing questions on the board about the example and allow a few students to give possible answers for those questions if time permits (2-4 minutes). If time does not permit, just tell students to keep those answers in their mind as the class goes on.
- Objective(s):
  - Determine the slope of a linear graph
  - Compare functions of different forms with the same input
- Key Terms: input (x), output (y), y-intercept, slope
- Explain purpose and use of concept:
  - No matter how data is presented, you can compare it
- Relation to past learning:
  - Inputs and outputs
  - Relation of tables, equations, and graphs with functions
  - Use of y=mx+b
  - Translating word scenarios and other functions
- Intriguing Questions:
  - What are the different ways of identifying rates of change in different forms of functions?

  - How might you identify a slope in a word scenario?

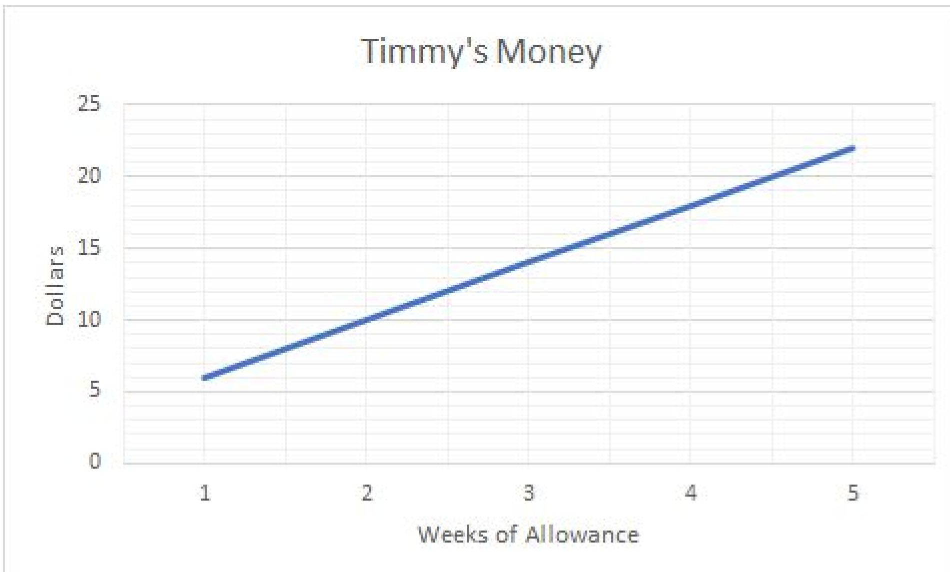
- Example(s):
- Determine the slope of the given graph.



-

Guided Instruction (10-15 minutes):

- Present following problem to class on board
- Ask class how they may solve the problem and give 1-2 minutes for class to think.
- Then allow students to discuss their proposals on how to solve and encourage other students to ask questions and think of other ideas as well.
- If students get stuck use prompts and ask guiding questions when needed.
- Allow as many ideas as time permits, but highlight ideas that connect well to previously shared ideas and the learning objectives.

- Example:Using the slope from the graph we just looked at to determine which of the following functions has the highest slope and which has a higher output for an input of 6.
  - y=6x-2
  - James starts out with 16 cookies, and he gives two cookies to each of his friends.
  - 

  -

Input	1	2	3	4
Output	2	4	6	8

- Prompt(s):
  - Think about what the output is and how it is changing.
- Questions to promote thinking:
  - How do you determine the slope of each function?

Collaborative Learning (10-15 minutes):

- Activity Layout:
    - Divide students into groups of 3-4 (preferably 4).
    - Number students in each group 1-4 or 1-3 if needed.
    - Have each student take a form or function that either has a higher or lower slope and/or a higher or lower output at a certain common input than another student in their group. Give instruction to each number separately, so they are not confused.
    - There will be no sharing as a class, just walk around to check-in on each group.
    - They get 10 points for participation.
    - For groups of four, follow these instructions:
        - Student one will make a word scenario with a higher slope than student two, but a lower slope than student four.
        - Student two will make an equation with a lower slope than student one, but a higher slope than student three.
        - Student three will make a table with a lower slope than student two, but has a greater output at input 6 than student four.
        - Student four will have a graph with a higher slope than student one, but will have a lower output at input 6 than student 3.
    - For groups of 3, follow these instructions:
        - Student one will make a word scenario with a higher slope than student two, but a lower slope than student three.

- Student two will make an equation with a lower slope than student one and higher output with an input of 5 than student 3.
- Student three will make a graph with a higher slope than student 1, but a lower output for an input of 5 than student 2.

Independent Learning (10 minutes or however much time is left):

- Procedure:
    - Present the following problem on a small handout and have students turn it in when done or the next class period. Have students write out work on a separate piece of paper. Students must apply at least 3 attempts to get full credit for each question (more than 3 attempts is okay). After each attempt students must explain what they learned from that attempt and what they will try differently for the next attempt. After their final attempt they must say why they think their answer is correct. Before they leave make sure they get at least one attempt for the question, so they can ask questions to get clarification before they leave.
    - Move around the room to check-in on students to see if they are struggling or not.
    - Grading system: 2 points per attempt, 2 points per explanation after each attempt, 2 points for accuracy for a total of 14 possible points.
- Problem: Imagine you have a line that is developed by two points A=(0,0) and B=(_,_). You also have a word scenario that says: A person has $100.00. Each

day they have $_ (can be positive or negative) added to/taken from their total. Using only the digits -9 to 9 to fill in the 3 blanks, make it so the line with points A and B has a slope that is larger than the word scenario's slope by the greatest possible amount. You may use each digit once per attempt.

- Answer: $-9 and B=(1,9)

Lesson #5 Explanation

For my fifth lesson plan I had two objectives and four key terms. My objectives were "determine the slope of a linear graph" and "compare functions of different forms with the same input". I chose the first one because in the previous lesson the students will have learned about the general idea of slope and how it relates to a linear function. The next step is explaining the slope in greater detail and helping them identify where it comes from and how to determine it. The previous lessons have also introduced several different forms of functions such as tables, equations, or graphs. Because of this, I chose an objective where students compare these different forms and can see how they are similar and relate. I also chose to keep the same key terms from the previous lesson because lesson #5 is just going into more explanation and developing a deeper understanding of these terms. In particular, slope is being explained in much greater detail. The other key terms are used to help the understanding of slope.

For the activity of the focused instruction the teacher will show a linear graph and explain the steps to determine the slope. This includes the slope formula and the rise over run method of counting the spaces up and to the side. I chose this activity as a brief introduction on the basic information on how to determine the slope. This allows them to have some understanding of how to find the slope of a graph before comparing the slopes of other forms of functions. The intriguing questions I chose were “what are the different ways of identifying rates of change in different forms of functions?” and “how might you identify a slope in a word scenario?” I chose these questions because I want students to think about how slope might work in other forms of functions such as tables or word problems. This gets them thinking and wondering about how the slope of a graph connects to the slope of its table or equation. Being able to connect the slope of a function between its different forms helps a student understand what the slope is on a deeper level.

For the guided practice I had to choose a problem for the class to walk through. For the problem I chose four different forms of functions. These forms include a graph, table, equation, and word scenario. The class is supposed to determine which function has the greatest slope and greatest output with an input of six. I chose this problem because it helps students have a side by side comparison between several different forms of functions. This allows them to explore and visualize what slope means in the context of different forms of functions. In addition, students can see more clearly that each function form is showing the same type of information. The only difference is its appearance. It also helps students learn how to identify what the slope is from tables, equations, graphs, and word scenarios.

In addition to the problem for the guided practice, I also had to choose a prompt and guiding question to ask the class. For the prompt I chose to tell students to “think about what the output is and how it is changing”. This encourages students to focus on identifying patterns to help determine the slope. The question I chose to ask was "how do you determine the slope of each function?" I chose this question because this problem is large and complex when looked at as a whole. It will be more beneficial for students to take it piece by piece and work through the process of finding the slope for each function individually. Then, afterwards they can more effectively compare the different functions to each other and see how they all connect and relate to each other.

For the collaborative learning phase of lesson #5 the different groups each have to create several different functions with certain characteristics smaller or greater in respect to each other. Each function used takes a different form of function. This problem is very good for collaborative learning because it has many different answers and strategies that can be used to produce a final product. Students get another chance to create something and take ownership of their learning in this activity as well. It also provides good practice for students to compare the slopes of functions in different forms. This problem also takes critical thinking for groups to create a group of functions that fit the criteria of the problem. Therefore, it helps students practice perseverance by causing some groups to attempt the problem multiple times.

For the independent learning section there is one Open Middle problem. In this problem students have to determine a slope of a word scenario and the x and y values of a point that when connected to the point (0,0) creates a line with a higher slope than the word scenario. They can only use digits between -9 and 9 and each digit can only be used once per attempt. I chose this problem because it calls for multiple attempts by the students. This allows them to not only feel free to make mistakes, but it also encourages them to learn from their mistakes. This promotes critical thinking while a student tries to determine their strategy for their next attempt. This problem is a good example of comparing graph and word scenario slopes. As they explore how the different values affect the slope of each function they gain a better understanding of how slopes work for both functions in graph and word form.

Bibliography

Anderson, R. (2007). Being a Mathematics Learner: Four Faces of Identity. *Mathematics Educator*, *17*(1), 7-14.

Baker, Jessica L. W., (2017)"Growth Mindset and Its Effect on Math Achievement". Capstone Project and Master's Theses. 133.

Boaler, J. (2016). Designing mathematics classes to promote equity and engagement. *The Journal of Mathematical*

Boaler, J. (2016). *Mathematical mindsets: unleashing students potential through creative math, inspiring messages, and innovative teaching*. San Francisco, CA: Jossey-Bass & Pfeiffer Imprints.

Boaler, J. (2009). *What's math got to do with it?: how parents and teachers can help children learn to love their least favorite subject*. New York: Penguin Books.

Coherence Map, Retrieved April/May, 2020 from achievethecore.org/coherence-map/.

DeSilver, D. (2017, February 15). U.S. academic achievement lags that of many other countries. Retrieved April 30, 2020, from https://www.pewresearch.org/fact-tank/2017/02/15/u-s-students-internationally-math-science/

Fisher, D., & Frey, N. (2014). *Better learning through structured teaching: a framework for the gradual release of responsibility* (2nd ed.). Alexandria, VA: ASCD.

Kaplinsky, R. (2020). *Open middle math: problems that unlock student thinking, grades 6-12*. Portsmouth, NH: Stenhouse Publishers.

MORRIS, J. (1981). Math Anxiety: Teaching To Avoid It. *The Mathematics Teacher, 74*(6), 413-417. Retrieved April 23, 2020, from www.jstor.org/stable/27962523

Smith, M. S., & Stein, M. K. (2018). *5 Practices for orchestrating productive mathematics discussions*. Reston, VA: National Council of Teachers of Mathematics/Corwin Mathematics.