Stack a Deck by Shuffling

Keri Osborne
Western Oregon University, kosborne16@mail.wou.edu

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The Goal

We wish to discover if we are given a traditional 52-card deck of playing cards, in new deck order, can we stack the deck by simply shuffling?

Our goal is to stack the deck for a fictitious game of five card draw poker, using a combination of the perfect shuffle, and variations of the perfect shuffle. So, we want a royal flush, which is the best hand in poker, to be sitting on the top of the deck when we are done. A royal flush is the Ace (A), King (K), Queen (Q), Jack (J), and 10 of the same suit. The royal flush is worth the same, no matter what suit it is in, so it does not matter which suit our royal flush is in.

When the deck is face down, we will let new deck order, from top to bottom, be A-K of Hearts, A-K of Clubs, K-4 of Diamonds, and finally K-A of Spades.

For our fictitious game of poker, we are going to make the big assumption, that when we shuffle a deck using the perfect shuffle, we will get a new deck order, can we stack the deck by simply shuffling?

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Perfect Shuffle Variations

The following are explanations of general forms of the in and out shuffle, that we will use to stack the deck.

For the R-carded out-shuffle begin with a deck of size m such that m = R \cdot d.

- If d is even, simply split the deck into two equal parts. Then, shuffle with R cards at a time starting with the bottom part of the deck.
- If d is odd, split the deck so that the top part of the deck has R more cards than the bottom part of the deck. Then, shuffle with R cards at a time starting with the top part of the deck.

For the R-carded in-shuffle begin with a deck of size m such that m = R \cdot d.

- If d is even, split the deck into two equal parts. Then, shuffle with R cards at a time starting with the top part of the deck.
- If d is odd, split the deck into two parts such that the top part of the deck has R less cards than the bottom part. Then, shuffle with R cards at a time starting with the bottom part of the deck.

R-Carded Shuffle: Suppose we have a deck of size m such that m = R \cdot d for R, d \in \mathbb{Z}. Let the card positions be numbered 0, 1, \ldots, m−1. Then, after one R-carded shuffle, a card in position x goes to the position given by the function s:

s(x) = \begin{cases} 2x \pmod{m} & \text{if } 0 \leq x \leq m−1 \\ x & \text{if } x = m \end{cases}

Note that the function s is either the function g for out-shuffles or the function h for in-shuffles depending on whether you are doing an R-carded in-shuffle or R-carded out-shuffle [2].

Fig. 1: The Perfect Shuffle.

Results

Now, the fun part. Let’s shuffle to get a royal flush at the top of the deck! For the purpose of this project, we are going to choose the suit of our royal flush to be Clubs. Note that initially the A of clubs is in position 13, and the 10, J, Q, K are in positions 22-25 respectively.

First, perform one, 4 Carded In-Shuffle using all 52 cards, so m = 4 \cdot 13. This will put the Q and K in positions 0 and 1, since s(\frac{13}{4}) = 0 and s(\frac{13}{4}) = 1. Similarly, the 10 and J are now in positions 46 and 47, since s(\frac{13}{4}) = 46 and s(\frac{13}{4}) = 47. Finally, the A will be in position 29, since s(\frac{13}{4}) = 29.

What’s Next?

Here are a few questions I would like to explore to further the exploration of stacking decks by shuffling.

1. How would you stack the deck if the dealer deals “correctly” by alternating players?
2. Can we stack the deck, while still choosing Clubs as our suit, in fewer shuffles? How?
3. Can we stack the deck, choosing a different suit, in fewer shuffles?
4. Can we stack the deck for other forms of poker, where players aren’t dealt five cards? What about other games that use a traditional deck of cards, such as Cribbage?

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References