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Research Based Best Practices for Teaching Mathematics and Improving Math Attitudes

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By

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Dedication

Thank you to my Lord and Savior, Christ Jesus, for showing me what He wills for me and allowing me to trust in Him for strength (James 1:2). Thank you to all my family and friends for supporting me in my journey to becoming a math teacher and bilingual educator. I have truly found my passion in life and can't wait to help others enjoy mathematics and see it for enjoyment rather than as a tool. Thank you to Breeann Flesch for your patience and wonderful ideas as we bounced our math education ideas and perspectives of each other in the writing of this thesis.

The Problem

Personal Narrative

This thesis has been inspired majorly through my experiences in the blend of mathematics and education through my eyes as a teacher and a student. I have generally enjoyed math. There have been struggles and eye-opening, amazing discoveries. I distinctly remember sitting in Pre-Algebra class in eighth grade and learning the equation of a line. It was so amazing to me. The idea that math was more than just addition, subtraction, multiplication, division, and fractions was so fascinating to me. This was a whole new way of learning math that connected to something tangible and real instead of how many apples we can put in some imaginary person's basket. My peers generally didn't find this magnificent at all. They struggled with it because of how new and different it was to the memorization of tables or flash cards that we had done all the years before.

In high school Algebra, I struggled with factoring and I regret not trying harder to grasp it. When I look back now, I realize that the pressures of my friends made me not try in my math classes in high school. My peers acted so cool and uncaring in math class but no other classes. I look back now and think that this was a coping mechanism because they were confused and math wasn't taught to them in interesting ways. They talked about how much they disliked it. So I did the same. Sure, I had a knack for math, but that didn't mean that I had to try or let others see it. I bought in to the idea that math was lame because I wanted to fit in. If I had tried harder and not cared about what my peers thought about math I would have taken AP Calculus with the "lame" teacher and “nerdy” kids rather than Statistics and Probability with the cool teacher and popular kids. I had the chance to expand my knowledge of a topic I was interested in, but I went the easier route to fit in.

I observed this negative attitude towards math continue even in my college schooling. In pre-service teacher math classes at Western Oregon University, many of my peers complain that they shouldn’t have to take the block of three basic math classes because they want to teach elementary school. I have heard them say many times that math is too hard or boring. What I can’t believe is that these are future teachers who have already decided math isn’t worth their time. How will they teach math in the elementary level if they dislike it and ignore the math classes they took?

As a homework grader for these math classes for future teachers, I have been appalled to see how some of the simplest things that students should have learned years ago have been neglected. For example, I once graded papers where students were using the area formula for a triangle to find the areas of a shape made of different triangles on a geoboard. In this problem, more than half the class made mistakes in their multiplication and addition. This is understandable for an elementary school student who is new to this material. However, for an adult going to college in order to teach elementary school to mess up repeatedly with their basic math skills is unacceptable. They also didn’t know how to properly label the area with units. This is a math skill that is used frequently in the real world. When we buy flooring for a home, for example, we need to know how to do the multiplication correctly and label the units properly. There is a huge difference between feet, square feet and cubed feet (length, area, and volume respectively).

In my opinion, this isn't only the students' fault. Not only do I think the blame falls on their teachers who taught these students math but it also falls on our society. There is a bigger problem here than not spending enough time teaching mathematics or not teaching it a certain way. The larger issue is how we are viewing math. Our perceptions affect how we talk, think, and interact with others. Our future teachers are approaching math with aversion. If they are doing this now, they may avoid teaching it too. A teacher's perception can rub off on his or her students.

If we pause for a minute to think about our cultural perceptions of mathematics in the US, we can see how we view this subject in a different light than all other subjects. If I told someone that I hate math, it would be rather normal. I can complain about how hard it is and say my brain just isn't wired to understand it, so I will give up trying. This is culturally acceptable. However, if I instead said those same things about reading it would not be culturally permissible. To admit that I hate to do mathematics is fine, but saying I hate reading and give up trying to read is unheard of in our culture.

This rejection of math is so deep within us and our society that the solution cannot just be to teach math in new and innovative ways. These are good things, of course. I include some in my set of best practices mentioned later. However, we need to find ways to address this societal issue of our math perceptions in order for these teaching practices to work well in the classroom.

While my experiences she light onto how the US may perceive math, I want to research this math education issue to find out more. Through research on the math perceptions of our society, I establish whether there is a problem or not with math education in the US. From this, I find solutions to the two problem areas of culture and teaching.

I will seek answers to questions like the ones below through research.

- Why are so many students avoiding math?
- What should our education system be doing differently?
- What should our society be doing differently?
- How can teachers be teaching math differently to improve student attitudes and content knowledge understanding?

If I can answer questions like these, I can get to a fundamental cause of the math education problem. This will allow me to propose solutions culturally and in the classroom. By investigating what others have said around the world about the math education situation, I can confirm my ideas and find out what needs to be done to improve math education in the United States. It is important to improve math education because as we can see from the past ten years, we have a high need for personnel trained in mathematics and other STEM fields to support the exponential growth of technology.

Opinions on Math Education from Journalists and Mathematicians

Despite the high need for many workers in STEM fields, it is true that people in the United States exhibit a severe avoidance to math and STEM professions.

Professionals who discus the importance of math education and propose solutions will even admit that they are afraid of math! In the middle of explaining an interactive, online math technique she learned from math experts, journalist Jessica Lahey says that "even I, an admitted math phobic, had a lot of fun playing on this site" (2015). This may seem like she is just trying to make the math activity sound fun and easy, but she is also acknowledging that it is acceptable to be fearful of math. Another writer said: "I'll admit that I'm one of those people who harbors an aversion for math". This author is in the middle of saying "our future … depends on making math more palatable" (Murphy Paul, 2013) yet makes it non-palatable and unpleasant by admitting an aversion to math.

These may not be mathematicians, I will admit, but they are advocates for an improvement of our math education system while at the same time they admit they have a fear towards math. These sources are supposed to be those who know the problem and wanted to help, but even they themselves are unable to whole-heartedly like mathematics. It is rare to hear an advocate of science, history, writing, or reading talk about how much they are scared of the subject in the middle of proposing a solution to improving that field. The author above calling for math to be more "palatable" goes on to describe that we don't like math because of fear. "People dislike math because they don't understand it, and this lack of understanding makes them feel incompetent and vulnerable." (Murphy Paul, 2013). This may be the case.

When Lahey admitted her phobia for mathematics previously, she was making it acceptable to openly be a math phobic. Why is it okay to say these things about math but not about other school subjects?

One mathematician describes this social acceptance to hate math in a unique way:

> "Each time I hear someone say, "Do the math," I grit my teeth. Invariably a reference to something mundane like addition or multiplication, the phrase reinforces how little awareness there is about the breadth and scope of the subject, how so many people identify mathematics with just one element: arithmetic. Imagine, if you will, using, "Do the lit" as an exhortation to spell correctly." (Suri, 2013).

This quotation doesn't just show that it is socially acceptable in our culture to talk about math in a narrow scope but not acceptable to talk that way about other subjects. It also gives a peek into the way our culture perceives mathematics. From Suri's description, it seems that most people see math as shallow, only containing simple arithmetic functions. It could be possible that there is a large gap in perceptions of mathematics between mathematic professionals and average people. Since we have established that the everyday person thinks rather negatively about or even fears math, it makes sense that this avoidance would lead to a shallow knowledge of the subject.

We see through each of these accounts that people are really quite scared of mathematics and hesitant to find out more about it. As a result, many avoid the subject in school and daily life. This leads to a huge portion of the population not understanding much about mathematics. For example, when a high school teacher "asked one of her geometry classes what 8 x 4 was, no one could come up with the answer without going to a calculator" (Welsh, 2012). According to these sources, this inability to do basic mathematics comes from the fear and anxiety that students have when approaching math.

Along with the fear of mathematics, “Americans have no respect for math” (Collins, 2008). Collins goes on to describe that *The Notices of American Mathematic Society* found that we have no respect for the field of mathematics and that this could be a reason that many of Americans do not enter this field. “The United States fails to encourage its students in math, fails to identify and encourage kids who could become the world’s top scientists and engineers, and the few kids that do succeed are almost all immigrants or the daughters of immigrants from countries that don’t consider math experts ‘nerds’” (Collins, 2008). If we don’t respect math as a topic of study nor see any value in it, then we are doing as this study depicted. We are holding our children back from becoming great people who discover new things. It seems that other countries don’t look at math experts as negatively as we do. When those people are respected, the field itself will gain equal respect too.

These attitudes of fear and disrespect for the field of mathematics combine to make it a subject that many people avoid. It seems that these sources agree that there is something about math specifically that turns away students and adults. Suri and Murphy Paul say it is because they don’t know enough about math. This could be true.

Jennifer Suh believes that it isn’t the lack of understanding but what happens before even that. This teacher’s students showed signs of “math avoidance and even math phobia” when she began teaching at the beginning of the year. “They saw mathematics as confusing and irrelevant to their lives; most disheartening, they associated it with failure." (Suh, 2007, p. 163). This personal account of an elementary school teacher shows that it isn't just adults who are fearful of math. Anxiety, phobia, and avoidance start early. The avoidance attitude towards math seems to be recurring in this source as well. However, this teacher sees it as a result of the students' feeling that math had no purpose to them. The lack of a purpose in math class led to avoidance and feelings of failure.

More evidence of discomfort comes from the story of a summer school teacher. Patrick Welsh accounts this teacher's experience and views of the students. The teacher feels that we push kids to move on to quickly in math before making sure they have the basics. As a result, "they get overwhelmed. Eventually they give up" (Welsh, 2012). So our students give up in mathematics because they are overwhelmed by the pressure to keep going onto the next class. If they are just passing Algebra I with a D grade and moving onto Algebra II as Welsh describes, they may not have enough pre-requisite information to thrive in their new Algebra II class. This is just digging students a deeper and deeper hole that they can't get out of.

These two personal accounts by teachers show that it is common for students to feel stunned and unmotivated when doing math. Feelings like these can make it even harder to teach mathematics when joined with the negative attitudes that our society upholds. This brings societal pressure to dislike math and makes it easier for the student to give up when it is an acceptable thing to do.

On top of the general societal pressures and a student's own feelings of being confused and overwhelmed by math, the students' culture and their age range doesn't allow for a positive view of math. When going over data posted in *The Notices of the American Mathematics Society*, Collins depicts how seriously uncool math is to do outside of school:

> "When asked why, a typical response is, "Only Asians and nerds do math extracurricularly." In other words, it is deemed uncool within the social context of USA middle and high schools to do mathematics for fun; doing so can lead to social ostracism. Consequently, gifted girls, even more so than boys, usually camouflage their mathematical talent to fit in well with their peers" (2008).

Teens of the US put their social status above learning math. The brightest students in math in the United States are the ones recently immigrated from China, Russia, Romania, and Korea. They are taught "rigorous essay-style proofs" at "math camps" while most teens are deeming themselves too cool to do math (Collins, 2008).

Students often question the purpose of math. On mathematician encourages us to "think of it this way: you can appreciate art without acquiring the ability to paint, or enjoy a symphony without being able to read music. Math deserves to be enjoyed for its own sake, without being constantly subjected to the question, "When will I use this?" (Suri, 2013). Suri believes that students could appreciate math for its beauty. This is easy for a mathematician to ask but hard for a group of high school students to grasp. It contradicts what Suh said previously that students don't understand and are confused by math and therefore they aren't interested in it enough to learn. What both seem to be calling for is some sort of intrinsic motivation to do math rather than being forced.

One researcher calls for this intrinsic motivation through what is called Recreational Mathematics. Sudoku and kakuro are examples. "The person has to want to solve the task with positive motivation, the solution process has to be viewed as joyful, and the product … at least somewhat interesting and possibly fun" (Sumpter, 2015). Intrinsic motivation, the author goes on to describe, is the best way to get students interested in math. All sources so far have not reported United States students to say math is fun and entertaining. When students were asking the question Suri posed, they were looking for a reason to practice mathematics. If teachers can't give them a viable reason and the student isn't intrinsically motivated, they will have no reason to be interested in mathematics.

Sumpter compares math content standards of prominent countries and/or those succeeding in math to find if they have anything written on positivity, pleasure, or enjoyment of mathematics. The researcher found that many countries did have something about attitudes or feelings of the students towards mathematics; except for the United States. The "Common Core Standards for Mathematics … focus on the use of mathematics" and see it as a tool rather than focusing on "the personal experience of using it" (Sumpter, 2015). When our standards don't ask teachers to encourage students to enjoy math, as so many other countries do, then the teachers don't have to make that a priority. Priorities easily shift elsewhere to passing tests and memorizing algorithms.

Sumpter's research along with the findings of many others has shown vast differences between the United States and Japan. The Japanese curriculum begins by explaining the importance of math enjoyment and the "merit of mathematical manipulation" (Sumpter, 2015). This contrasts severely with the complete absence of mathematical enjoyment in US curricula. Along with this finding on math perceptions and attitudes of Japan versus the US, there has been much debate in the mathematics education world over what Japan and other Asian countries are doing that causes them to score higher than the US year after year.

What people want to know is how the US can learn from countries that are excelling in mathematics. A National Council of Teachers of Mathematics (NCTM) article uses a study called the TIMSS which will be discussed in detail later. This test reflected similar findings that the United States sees math as a set of procedures and a tool to find an answer. Japanese teachers and students view math as a relationship of "concepts, facts, and procedures" (Hoffman & Brahier, 2008, p. 413). The way that the two cultures view math can greatly influence the way it is taught, interpreted, and practiced.

Elizabeth Green discusses the differences between the two countries similarly to Hoffman & Brahier. She distinguishes the opposite teaching patterns. The US teaches to the pattern "I, We, You" which begins with the teacher explaining a math concept. It is followed by the class working together on the procedure based lesson and then ends with the students practicing individually. Japan does exactly the opposite. Green calls it "You, We, I" and explains that it starts with a "problem of the day" for students to work on and play with. This is called "sense-making." They then proceed to talk together about it and then end with any needed teacher clarification or summation (Green, 2014).

Elizabeth Green's article posted in the New York Times in 2014 has stirred up much debate in the mathematics education community. Her article follows the story of a Japanese man who came to the United States because of the amazing publications by the NCTM on education reform. He saw the US as the center of mathematics but was sadly wrong when he found out that the progressive and innovative teaching methods published by the NCTM and others were not actually being implemented in the classroom. She blames the US for not implementing teaching reforms and not training teachers enough. Green summates that this is the difference between the US and Japan. Japan has implemented progressive teaching reforms and the US is still stuck in the traditional way of doing things.

In response to her article and its controversy, Notices *of the American Mathematical Society* asked various mathematicians to respond to Green. Those disagreeing with her believed the problem stemmed from elsewhere. For example, Hung-His Wu , a professor at UC Berkley, sees the issue with the US math system to lie in textbooks (Saul, 2015, p. 509). Guershon Harel, a UC San Diego professor, calls for "practice with understanding" (Saul, 2015, p. 514). This disagrees with Green who basically shunned repeated practice and drilling. Harel sees practice as an important way for students to "internalize, organize, and retain concepts and ideas" as long as the students understand why they are doing the math the way they are.

Hyman Saul, a professor at the University of Michigan, responds in agreement with Green. He agrees that the Common Core State Standards (CCSS) are promising. However, he goes on to explain that curricula, standards, and assessments are easy to achieve. It is professional development that is more difficult and expensive to accomplish and is usually left behind when the new education reform comes along (p.508-509).

The purpose of discussing this Elizabeth Green article and its responses is to show how varied opinions are on math education reform. Some see the problem lying in textbooks while others see the problem in teacher training. Sources before the Green discussion see the problem in math attitudes, feelings, or lack of enjoyment and motivation. There are so many opinions on where the blame lies in the US math education system but one thing is in agreement, something is wrong.

The Data

The Trends in International Mathematics and Science Study, previously known as the Third International Mathematics and Science Study, (TIMSS) looks at 4th and 8th graders in about 60 different countries and their skills in mathematics and science (Mullis, Martin, Foy, & Arora, 2011, p. 5). This study has the goal of "helping make informed decisions about how to improve teaching and learning in mathematics and science" (Mullis, Martin, Foy, & Arora, 2011, pp. 25-26). By collecting worldwide data, a country can see where they are in comparison with their competitors. When we do this for the United States, we see that the US is above the centerpoint of 500 for both 4th and 8th grades in mathematics.

This would point towards a conclusion that the US is doing well in mathematics. However, a deeper analysis of the data is needed to conclude whether the US is really exceeding in mathematics or not. Simply being above the average doesn't mean a whole lot. Also, why should the US settle for just being average? Why not try to score higher?

Figure 1 to the right shows that the US scored 541 in Mathematics Achievement in 4th grade for the 2011 TIMSS. The leader, Singapore, is 55points above the US. This puts the United States in eleventh place out of the fifty-one countries represented in this data. This might seem like good news, but, the countries the US is beating are generally poor, war torn, or small. The only other county that would be similar in size to the US that scored lower than the US is Germany. All other economic competitors with the US are scoring better than the US in math proficiency of our fourth graders.

Country	Average Scale Score
Korea, Rep. of	613 (2.9)
2 Singapore	611 (3.8)
Chinese Taipei	609 (3.2)
Hong Kong SAR	586 (3.8)
Japan	570 (2.6)
2 Russian Federation	539 (3.6)
3 Israel	516 (4.1)
Finland	514 (2.5)
2 United States	509 (2.6)
‡ England	507 (5.5)
Hungary	505 (3.5)
Australia	505 (5.1)
Slovenia	505 (2.2)
1 Lithuania	502 (2.5)
TIMSS Scale Centerpoint	500
Italy	498 (2.4)
New Zealand	488 (5.5)
Kazakhstan	487 (4.0)
Sweden	484 (1.9)
Ukraine	479 (3.9)
Norway	475 (2.4)
Armenia	467 (2.7)
Romania	458 (4.0)
United Arab Emirates	456 (2.1)
Turkey	452 (3.9)
Lebanon	449 (3.7)
Malaysia	440 (5.4)
1 Georgia	431 (3.8)
Thailand	427 (4.3)
ψ Macedonia, Rep. of	426 (5.2)
Tunisia	425 (2.8)
Chile	416 (2.6)
ψ Iran, Islamic Rep. of	415 (4.3)
ψ Qatar	410 (3.1)
ψ Bahrain	409 (2.0)
ψ Jordan	406 (3.7)
ψ Palestinian Nat'l Auth.	404 (3.5)
ψ Saudi Arabia	394 (4.6)
ψ Indonesia	386 (4.3)
ψ Syrian Arab Republic	380 (4.5)
ж Morocco	371 (2.0)
ψ Oman	366 (2.8)
ж Ghana	331 (4.3)

Figure 2: Distribution of Mathematics Achievement for 8th grade (Mullis, Martin, Foy, & Arora, 2011, p. 42)

Country	Average Scale Score
2 Singapore	606 (3.2)
Korea, Rep. of	605 (1.9)
2 Hong Kong SAR	602 (3.4)
Chinese Taipei	591 (2.0)
Japan	585 (1.7)
‡ Northern Ireland	562 (2.9)
Belgium (Flemish)	549 (1.9)
Finland	545 (2.3)
England	542 (3.5)
Russian Federation	542 (3.7)
2 United States	541 (1.8)
‡ Netherlands	540 (1.7)
2 Denmark	537 (2.6)
† 2 Lithuania	534 (2.4)
Portugal	532 (3.4)
Germany	528 (2.2)
Ireland	527 (2.6)
2 Serbia	516 (3.0)
Australia	516 (2.9)
Hungary	515 (3.4)
Slovenia	513 (2.2)
Czech Republic	511 (2.4)
Austria	508 (2.6)
Italy	508 (2.6)
Slovak Republic	507 (3.8)
Sweden	504 (2.0)
2 Kazakhstan	501 (4.5)
TIMSS Scale Centerpoint	500
Malta	496 (1.3)
‡ Norway	495 (2.8)
2 Croatia	490 (1.9)
New Zealand	486 (2.6)
Spain	482 (2.9)
Romania	482 (5.8)
Poland	481 (2.2)
Turkey	469 (4.7)
2 Azerbaijan	463 (5.8)
Chile	462 (2.3)
Thailand	458 (4.8)
Armenia	452 (3.5)
1 Georgia	450 (3.7)
Bahrain	436 (3.3)
United Arab Emirates	434 (2.0)
Iran, Islamic Rep. of	431 (3.5)
2 Qatar	413 (3.5)
Saudi Arabia	410 (5.3)
ψ Oman	385 (2.9)
ψ Tunisia	359 (3.9)
† ж Kuwait	342 (3.4)
ж Morocco	335 (4.0)
ж Yemen	248 (6.0)

Figure 1: Distribution of Mathematics Achievement for 4th grade (Mullis, Martin, Foy, & Arora, 2011, p. 40)

The eighth grade data (Figure 2) show similar findings. The US is ninth out of forty two countries. This might seem good, but in fact, our scores compared to fourth grade dropped by about thirty points. This drop from forth to eighth grade in test scores isn't as significant or isn't seen at all in the countries scoring higher than the US.

In both grade levels, the countries with the highest scores are Asian countries. The top five are Hong Kong SAR, the Republic of Korea, Singapore, Chinese Taipei, and Japan. Notice that even our non-Asian competitors like Finland and Russia are beating us. This points towards something that teachers of the exceeding countries are doing that maybe we are not and brings us right back to the different ways of viewing math as discussed in the previous section.

Along with math ability, TIMSS charts how students value mathematics. The students use what is called a Students Value Mathematics Scale to answer questions on their view of mathematics. In the 2015 student questionnaire, fourth grade students were asked to rate the following (and more like these) on a four point scale of agreement:

- I enjoy learning mathematics
- I wish I did not have to do mathematics in school
- Mathematics is boring
- I like any schoolwork that involves numbers
- I look forward to mathematics lessons
- My teacher lets me show what I have learned
- My teacher listens to what I have to say
- I know what my teacher expects me to do
- Mathematics is harder for me than many of my classmates
- My teacher tells me I am good at mathematics
- Mathematics makes me confused
  (TIMSS, 2015, pp. 15-18)

Staff is asked to explain their views and teaching strategies of mathematics as well. Principals are asked questions about math resources provided and whether they track their students. Teachers are asked how much time is spent on mathematics if they are an elementary classroom and how class time is spent. Questions also include:

- Are you confident in inspiring students in mathematics?
- Are you confident in making mathematics relevant to students?
- Are students permitted to use calculators?
- Do students use technology during their lessons?
- How often do students receive homework?

(TIMSS, 2015)

These three types of responders; students, principals, and teachers; all contribute to the TIMSS data to answer questions of the conditions of a math classroom and their attitudes towards the subject.

What TIMSS 2011 has found is a strong correlation between achievement and a student's confidence and attitude. When we look at students' responses to questions like these we find that there is "a strong positive relationship within countries between student attitudes toward mathematics and their mathematics achievement" (Mullis, Martin, Foy, & Arora, 2011, p. 19). This means that in countries where more students have responded that they like learning math on the questionnaire that went along with the test, those countries are much more likely to actually do well in math and are often the top in the achievement scores. The same follows for students who feel confident in math (Mullis, Martin, Foy, & Arora, 2011, p. 20). This shows how confidence and a positive attitude in mathematics correlate with achievement.

	Mean score in PISA 2012
OECD average	494
Shanghai-China	613
Singapore	573
Hong Kong-China	561
Chinese Taipei	560
Korea	554
Macao-China	538
Japan	536
Liechtenstein	535
Switzerland	531
Netherlands	523
Estonia	521
Finland	519
Canada	518
Poland	518
Belgium	515
Germany	514
Viet Nam	511
Austria	506
Australia	504
Ireland	501
Slovenia	501
Denmark	500
New Zealand	500
Czech Republic	499
France	495
United Kingdom	494
Iceland	493
Latvia	491
Luxembourg	490
Norway	489
Portugal	487
Italy	485
Spain	484
Russian Federation	482
Slovak Republic	482
United States	481
Lithuania	479
Sweden	478
Hungary	477
Croatia	471
Israel	466
Greece	453
Serbia	449
Turkey	448
Romania	445
Cyprus[1,2]	440
Bulgaria	439
United Arab Emirates	434
Kazakhstan	432
Thailand	427
Chile	423
Malaysia	421
Mexico	413
Montenegro	410
Uruguay	409
Costa Rica	407
Albania	394
Brazil	391
Argentina	388
Tunisia	388
Jordan	386
Colombia	376
Qatar	376
Indonesia	375
Peru	368

Figure 3: PISA 2012 mean mathematics scores (OECD, 2012, p. 5)

When we think back to the accounts of journalists, mathematicians, and others from the previous section we see that their beliefs and experiences are backed up by the research. There has been a lot of research about the complex phenomenon of motivation and its relationship to learning mathematics. For example, students' motivation to learn can be affected by whether they find the subject enjoyable and place value on the subject. In addition, students' motivation can be affected by their self-confidence in learning the subject" (Mullis, Martin, Foy, & Arora, 2011, pp. 326-7). Motivation and self-confidence in mathematics are so important to student success in the math classroom. More on these qualities of an efficient math classroom will be discussed shortly.

Another test, called the PISA (Programme for International Student Assessment) looks at fifteen year-olds abilities in mathematics, reading, and science. We look at the scores of these students to the left in Figure 3. This shows an even more saddening gap between the United States and the top countries.

The average score for OECD is a 494. The leaders are in the high five hundreds with Shanghai-China breaking six hundred at 613. China is marked with a yellow color to indicate that it is a country with students performing above average and holding some of the top performers in the world. The US is at least thirty-five countries below the leader and below the average by eleven points. Our country is colored blue to indicate that our students have performed below the average.

On the TIMSS, it was encouraging that even though we aren't among the top scores, we are at least above the average. For this test on fifteen year-olds, we fall below the average with smaller or less wealthy countries like Italy, Latvia, Slovenia, and others beating us by many points.

The PISA includes commentary much like the TIMSS. They too see the correlation between confidence and achievement but also add that "[students] whose parents have high expectations for them… tend to have more perseverance, greater intrinsic motivation to learn mathematics, and more confidence in their own ability to solve mathematics problems" (OECD, 2012, p. 18). Engagement is also one factor PISA takes into account. The analysis of the data explains that with a better teacher-student relationship comes higher engagement in the class.

Anxiety also is linked to mathematics achievement: "One way that a student's negative self-belief can manifest itself is in anxiety towards mathematics. Some 30% of students reported that they feel helpless when doing mathematics problems: 25% of boys, 35% of girls, 35% of disadvantaged students, and 24% of advantaged students reported feeling that way" (OECD, 2012, p. 18). So Welsch and Paul were correct in saying that we can have high rates of anxiety in math specifically.

From the TIMSS and PISA, we can see that the students of the United States are not scoring as well as competing countries. More importantly, we can see for the questionnaire data of these two tests that our students do not view mathematics class or mathematical learning positively. It seems that the opinions of our beginning commentaries align with the data.

Solutions

Change the Culture

The data is clear, Asian countries like Singapore, South Korea, Japan, and China continue to exceed the United States on math performance and their attitudes towards math (as cited in Clemmitt, 2015, p. 320). Hoffman and Brahier propose that the United States adopt some of the practices of these Asian countries to improve our test scores on tests like TIMSS and to develop mathematical learning. The authors recommend that the U.S. looks at the teaching practices of Japan superficially.

One drastic difference between these two countries is the format of a lesson. In the US, a typical lesson follows like this: 1. Review previous content by going over homework or doing a warm-up 2. Introduce new content by telling and demonstrating how to solve 3. Give in class practice problems similar to the one demonstrated 4. Check answers and assign homework 5. Demonstrate a different type of problem or let students finish homework (Hoffman & Brahier, 2008, p. 413). This lesson plan structure is guided and does not allow students to figure the math out for themselves. This is a prescriptive view of mathematics because students are told what to do and how to do it.

In contrast, a Japanese lesson structure looks almost reverse: 1. Review previous content by connecting an introduction of today's lesson 2. Present problem of the day for individual student work 3. Students work on the same problem in groups to compare solution methods 4. Teacher chooses a few students to present their solution to class 5. Teacher leads class in discussion of solution methods and summarizes and extends what students have contributed 6. Teacher summarizes lesson and presents new problem to practice methods used for the problem done in class (Hoffman & Brahier, 2008, p. 413). This lesson structure is still guided but is guided after students have had time to try to solve a problem. This allows them to foster their own mathematical thinking rather than procedurally doing what they are told. "Overall, Japanese teachers develop concepts more often that U.S. teachers, who tend to state the necessary steps" (as cited in Hoffman & Brahier, 2008, p. 413).

This lesson structure difference points towards the idea that these two countries' teachers view good mathematics instruction differently. U.S. teachers emphasize memorization of many definitions and procedures as the lesson structure reflects. Japanese teachers emphasize thinking about problems and the methods for solving rather than the right or wrong answer (Hoffman & Brahier, 2008, p. 414). This makes sense when the TIMSS data on math attitudes is considered. A 1999 survey showed that "in Japan, 73 percent of the teachers wanted their students to think about things in a new way, such as seeing different mathematics relationships. However, 61 percent of the U.S. teachers wanted their students to learn the skills needed to solve problems" (as cited in Hoffman & Brahier, 2008, p. 415). It is evident that in the Japanese culture, people value mathematical thinking more than in the U.S. and that the U.S. students and teachers value getting to the answer. As one journalist would explain it: Americans are obsessed with short cuts (Horton). "We expect our kids to compete with those in the rest of the world…but we want to do it without requiring as much total time studying and doing homework." (Horton). Nick Horton is saying that we need to teach our students how to work through productive struggle. The value is placed on the answers to many problems instead of the understanding of the problem and solution to the answer. The prescriptive procedure used in the U.S. lacks real-world application because understanding is not the focus (more on this later).

Hoffman and Brahier suggest the U.S. take on a lesson plan structure more similar to Japan's because the TIMSS data show that "a mathematics teacher should never "tell" students anything" (Hoffman & Brahier, 2008, p. 415). That is what the U.S. is doing, telling kids how and checking the answers rather than valuing the process to get the answer. TIMSS scores show that this is not working for the U.S. Changing the lesson plan structure is one step in changing the view that our country has on mathematics. Allowing students to reason through why the math works for themselves may help them to see value and purpose in mathematics. This in turn, according to the data, can increase the likeliness of better performance.

When this Japanese structure is applied to a math class, the teacher himself or herself must be willing and prepared for this new approach to teaching math. If the teacher is unwilling to let students explore a problem and solve it different ways, then this structure will not work. It cannot be forced. Embedded within this structure is student discourse and participation. The teacher must relinquish some control and allow students to share their ideas.

The purpose of implementing this structure is to bring students to discovering math for themselves and to engage them in deeper math thought and real world thinking. In the U.S. recently, we have seen a small increase in the textbooks to have this type of investigative learning that the Asian math teaching structures uphold. However, this "investigations" are in reality more of a project. Students may be thinking deeply and analyzing math by connecting it to the real world but they are doing this after having a portion of teaching instruction. While this strategy might be half-way there, it is nowhere near to helping our students to investigate math and develop their own understandings of how math works. More on this misuse of the idea of "investigation" will be discussed later in the lesson plan analysis.

Deeper Understanding

While implementing the Japanese lesson plan structure is ideal, it may not fit with the curriculum required by a given school or district. The following solutions can be great ways to increase depth of understanding in the math classroom. We want to increase our students' understanding by depth so that they can then see the interrelated concepts of mathematics and how math can branch out into other subjects as well. A shallow understanding is the procedural understanding of the US lesson structure method that only allows for memorization and math as a tool. This method, as we have seen, does not lead to a true understanding, appreciation of, or success in mathematics. These solutions can pair with the Japanese lesson structure as well as standalone as best practices when teaching mathematics.

Discussion

Traditional classroom discussion is "going over answers and mistakes" (Tanner, 1998). This is much like the US lesson structure and US view of mathematics: a tool and procedure. Discussion in a math classroom agrees with the Japanese lesson structure because it allows all students to interact with the math and go deeper into understanding it.

Having small group discussions allows all students to be active learners (Tanner, 1998). When students are all explaining their thinking to a partner through strategies like Think Pair Share or Turn and Talk (both ESOL strategies) they are all getting to say something about the problem. This contrasts with the traditional answer giving where a student raises their hand or is called on to explain, because that is really only allowing one student to reason through an explanation rather than all.

One activity Tanner mentions can be used in the classroom to promote discussion. The fifteen to twenty minute is set up do that there is an inner circle where each student has a certain amount of tokens which relate to the number of times they contribute. The outer circles students take notes on the discussion. The discussion begins with the teacher posing a question and ends with a summary of the discussion. After the discussion there is a debriefing, this allows all students to contribute, no matter which circle they are in. This strategy, that is often called a Socratic Seminar, can be used to discuss any math topic really. Tanner recommends it be used after reading a section of math text, however it can be used to explain a mathematical process or work as a team to reason through various ways of solving a given problem.

Writing

Writing brings depth of knowledge because it allows students to "become more than just efficient calculators" (Enyart & Van Zoest, 1998, p. 165). Students have an opportunity to show high levels of thinking when they write about their math work. A teacher can prompt students to write about a variety of things:

- Interpreting data from tables, graphs, or charts
- Explaining their solution process
- Explaining something they do not understand and why
- Writing a word problem or a series of connected math problems
- Describing how to solve a certain type of problem
- Researching how a certain math topic is used in a career path and reporting on how it works and why it is important

Each of these writing ideas can bring in new ways to deepen mathematical understanding because they are asking students to elaborate. When students use elaboration they are not just using a procedure and getting to an answer. Instead, students are interacting with the math at each step along the solution path to get to where they want to go. As students reflect through writing along the way to the answer they are proving their understanding to their teacher.

In the last suggestion on researching the way a math topic is used in a specific career, the math topic is being connected to a real life application. Not only is this topic showing how the math is used in the real world but it is showing the student what they can use this math for once they have mastered it. This is promoting students to go into careers that use mathematics because writing projects like these give "students some insight into the purposes and ways of mathematics." (Kline, 1971, p. 11)

By including writing in the math classroom students are deepening their knowledge as well as improving their writing and explanation skills. It could also be beneficial to communicate with English teachers, if in secondary schools, to find out what basics students are currently working on. If they are working on argumentative writing, for example, you can have students argue their claims or paths for solving.

Real Real-World Applications

There are many ways to make real-life applications between the math being taught and the math the standards want them to learn. One easy way to connect the math to the real world is by using situations from students' lives. For example, many students like video games; this could be used simply as numbers like twenty video games per box and five boxes which means there are one hundred video games. This way, however, is less purposeful because it is simply using the real application as numbers to then practice the math standard. Instead, students could be shown how math works within video games and programming or they could use a video game to show math. There is a movement online to use Minecraft, a popular computer game for kids, to teach and geometry concepts.

Real world connections can also be made across disciplines. "We must relate the mathematics to history, science, philosophy, social science, art, music, literature, logic as well as to any other development which the topic in hand permits." (Kline, 1971, p. 12) The trick to making this beneficial is to communicate with teachers of other subjects. While Professional Learning Communities are very popular, they are often separated by subject and grade level in secondary. While this is great for day to day, it is important to also communicate with other subject areas so that connections can be made across disciplines.

Other options are to share about yourself through mathematics, use popular culture, or other topics that interest the whole class. Another great strategy is to simply get to know the students and ask them what they like. When they learn about the things they like and how they connect to math, there will be a significant increase in their engagement and motivation.

No matter which type of real-world connection is chosen to be used in a math classroom, it is important to use it in the proper way. Real-world connections should be used in realistic ways that show the purpose in what students are doing. Yes, using something of interest can also increase engagement but there is true engagement when those things are used to show purpose in mathematics rather than engage interest by counting games rather than apples.

Seeing Mathematics Recreationally

Other than the way structure in which we teach math, we can also change our attitude. The way our country views math is so different than the way many others view it. Lovisa Sumpter decided to compared curricula and syllabi of nations across the world and found many differences in the curricula which can affect math attitudes and mathematical teaching approaches.

To start with, one curricular objective in Japan is that "'students will […] (3) recognize the joy of mathematical activities and the merit of mathematical manipulation'" (as cited in Sumpter, 2015, p. 127). As was evident when comparing Japan and United

States lesson structures, Japanese education values student reasoning and understanding of mathematical processes.

Singapore's syllabus for primary schools mentions enjoyment, like Japan's does, and also that the importance of this joy in math will lead students "'to pursue mathematics beyond the classroom walls'". This nation wants its students to see math as more than just an accademic pursuit to get a good grade. Singapore also focuses on using math creatively to grow "'confidence in and develop appreciation for the subject'" (as cited in Sumpter, 2015, p. 128).

Of course China, like all countries, wants their students to have solid mathematical understanding. China's curriculum calls for positive participation which includes mathematical curisoity, creativity, and enjoyment (Sumpter, 2015, p. 125).

The officials, politicians, and educators in the Japanese, Singaporean, and Chinsese cultures value mathematical understanding and appreciation. This is made clear in their curricular wording when they use words like appreciation, curiosity, eagerness, and enjoyment.

England highly values mathematics like these Asian countries, but values it in a slightly different way. Their Mathematics Programmes of Study state the importance of mathematics as "a creative and highly interconnected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems". The importance of quality mathematics instruction is that math is "a foundation for understanding the world, the ability to reason mathematically, an apreciation of the beauty and power of mathematics, and a sense of enjoyment and curisoity about the subject" (as cited in Sumpter, 2015, p. 125-126). English curricula focus on enjoyment as well as using math to understand the world.

In stark contrast, the United States does not mention enjoyment, appreciation, or anything about an individual interaction with mathematics in its curriculum, The Common Core State Standards for Mathematics. Sumpter describes that "there is no mention of the affective side of the subject." Instead, the U.S. curriculum mentions the "use of mathematics and how it is used but not the personal experience of using it" (2015, p. 129). This corresponds with Hoffman and Brahier's article comparing lesson plans of Japan and the U.S. because it is evident that the U.S. sees math as a prescriptive tool and not a relational process.

When we list these countries' test scores on the PISA and TIMSS next to a summary of their wording in their curriculum, we can see a relationship. This table is shown on the next page. Countries who have wording that values mathematics' beauty within their curriculum seem to do better on their tests. The argument for this is that because they value math's process, creative, and beuty in their curriculum, the teachers approach the way that they teach math with this as their goal for student understanding rather than that of a tool that must be learned. Japanese students learn math with deep understanding because their teachers value the reasoning and process of math. In the U.S., we teach memorization and handy formulas to solve math without placing value on students' understanding of the process of math.

One outlier is England. They scored fairly evenly with the United States even though they have similar wording to the three Asain countries. This could be that

England has recently been changing their curricular views so their test scores are not yet reflecting their new approach to math teaching. From what Sumpter has said in her article, it seems that the Mathematics Programmes of Study is rather new, which could mean that with this slightly older score data, the Programmes were not in effect yet.

Country	**TIMSS 8th grade scores 2011** (OECD, 2012, p. 5)	**PISA scores 2012** (Mullis, Martin, Foy, & Arora, 2011, p. 42)	**Curricular views** (Sumpter, 2015)
Singapore	611	573	Enjoyment, Beyond the classroom, Confidence, Appreciation
*China	609, 586	613, 566, 561	Enjoyment, Creativity, Positive attitudes and participation
Japan	570	536	Enjoyment, Process
England	507	494	Creativity, Appreciation, Foundation to understand and reason with world
United States	509	481	Tool, Resource
Average Score / CenterPoint	500	494	
*China is divided into two or three parts for these tests. For the sake of this discussion, it is considered under the same name because other sources on this topic call all two or three sections "China".			

It is clear in these syllabi and curricula that countries with respect for appreciation and enjoyment of math have better test scores on average. The U.S. curriculum does not address the importance of enjoying or appreciating mathematical understanding. "[Math] appreciation may provide more than just casual enjoyment: it could also help change negative attitudes toward the subject that are passed on from generation to generation" (Suri, 2013). Appreciating math may not seem important; however, it is vital to recovering our nation's attitude towards math which will in turn improve our test scores. Appreciation and enjoyment means that "[students] have a better chance of succeeding in a subject perceived as playful and stimulating, rather than one with a disastrous P.R. image" (Suri, 2013). A school subject with this "disastrous P.R. image", Suri explains, is mathematics first and foremost. It has a reputation for being disliked and avoided. Suri wants math to be engaging and interesting because then students have a better chance of succeeding in the subject of math.

This idea is frequently called recreational mathematics. Recreational mathematics requires positive and intrinsic motivation as well as enjoyment in the solving process (Sumpter, 2015, p. 122). When our students see how mathematics can be done recreationally, for entertainment, outside of the classroom (as some countries describe in their curricula) they will be able to see a larger purpose in doing math. This gives them motivation beyond external factors like getting good grades. Recreational interest allows students to have a stronger type of motivation, intrinsic motivation. This form of motivation comes from a personal desire to do math instead of being forced or motivated by an external reward. With an intrinsic desire to study mathematics, students are far more likely to pursue the subject beyond their required courses.

Many things can be done to bring recreational mathematics and other ideas of enjoyment from the Asian countries' curricula into the classroom. One solution is to bring intrigue into the math class: "kids love puzzles, and wrestling with good ones can help them fall in love with math and get stronger at it" (as cited inLahey, 2015). Bringing in problems that are interesting to solve and are challenging will increase a student's motivation to complete the task. This can make learning math fun and bring more depth to student learning since the problems are more complicated.

Suri suggests that when math is made more enjoyable in the classroom, there is less need for the ever constant question of "When will I use this?" (2013). He explains that students do not ask this for other subjects at the rate that they ask it in math class. Sometimes math should be enjoyed for its own sake (Suri, 2013). This is not to say that the purpose and use of learning shouldn't be given. It is very important to connect it to the real world. However, raising students' enjoyment in class by making intriguing lessons is definitely a solution that can improve attitudes and in turn math scores because with higher interest comes more learning.

To use puzzles in class is not to just give students a handful of random difficult math questions. A teacher must prepare specific math puzzles to pose for his or her students and prepare the students by setting expectations of work and mathematical reasoning. Tying this into the idea of the Japanese lesson plan structure, students are posed with an intriguing question and not given an exact procedure to follow. This encourages inquiry as they try different solution paths.

My Reflections

The data from the TIMSS and PISA scores show that the United States is not stacking up in comparison with its competitors. Our society talks about math negatively and does not view it as something with a beautiful process but rather a tool. The math education system has been failing for some time in the United States and no matter how many times NCTM or another organization comes out with a new teaching method we both fail to implement it or don't implement it at all (according to Elizabeth Greene).

I believe that our education system cannot be changed until we change our beliefs about mathematics. This starts with our society and culture. We need to value math not as a tool but as a process that has purpose in the way we get to an answer. It is not okay to continue passing down these negative attitudes to the next generation. To change our culture we need to change the way we perceive and talk about math. Instead of viewing math as a toolbox of procedures, we need to view it like other Asian countries do, as a process that leads to understanding of our world.

If our students have positive role models who support math as something more than memorization and formulas, they will want to go into STEM careers our country needs. If math is viewed as something interesting and creative, like writing of ten is, students will be engaged in their classes, get better scores, and be interested in taking math beyond just the minimum for graduation. This all comes with the caveat that our students aren't inheriting the negative attitudes of the previous generation. When we change our cultural perceptions of mathematics, only then can we fix our failing math education system.

Appendices

Common Core State Standards for Mathematics

Some may wonder where the Common Core falls amongst all of this. Are these standards a solution, anti-solution, or neither to the problem of mathematics education in the U.S.? There are many different opinions on the usefulness and effectiveness of the CCSSM in teaching students in a way that they have a deep and interconnected understanding of math.

States have had the option to adopt the CCSSM but many have been convinced into it because of the promise to get rid of the consequence of not meeting the requirements of No Child Left Behind. As of May 2015, 42 states have adopted, four states have not adopted, three states have withdrawn their adoption, and one state has made modifications in adoption (Academic Benchmarks, 2015).

Solution

After thorough research, little could be found that whole heartedly claims that CCSSM is the solution to math education problems except for from the writers and promotion from CCSSM website itself. The benefit of CCSSM is that it gives a unity to each subject taught across the country. For example, when a student moves state to state or even just to a school nearby, they theoretically (if states choose to adopt these standards) should no longer have the problem of having a teacher who didn't teach a certain topic the year before. Other than this way of being a solution, most other commentaries talk about its drawbacks.

Anti-Solution

As discussed previously, Sumpter read the standards and found it lacking statements on the usefulness, necessity of enjoyment, and appreciation of mathematics. She found that the U.S. does not value the personal experience of using math (2015, p. 129). With this in mind, her opinion is that it lacks many things that could greatly improve the country's perception of the subject.

CCSSM is technically not a curriculum; teachers still teach the content in the way they choose. However, while not a curriculum, it drastically narrows the possible curricula a state or district can choose (Horton). The Common Core lists a specific set of topics to be covered in a specific grade level. This means that the options are fewer to choose from when picking a textbook or worksheets. It is making teaching math harder because the good curricula no longer qualify because the topics are arranged differently or scattered among the grade levels. Horton believes that the Common Core is not targeting the correct problem, "Until we change who we are, no curriculum on earth will save us."

In-depth reports are given to congress as they make decisions in the government. CQ Researcher has a section from 2015 explaining the pros and cons. Some pros and cons are given but the one that stands out the most is that the Common Core is "the opposite of what teaching and learning thinking demands" and that it is a disrespect to teachers because it does not allow teachers to be creative (Clemmitt, 2015, p. 331). This goes against my last solution, developing a deeper understanding of mathematics, because the Common Core asks for memorization lectures rather than "thinking-oriented class", says Clemmitt.

Abstract for the Academic Excellence Showcase, May 2016

Mathematics education is at risk in this country. Not only are we falling behind in international math tests, but our citizens have negative math attitudes and are even math phobic. In this presentation, we investigate solutions to this problem through research and propose best teaching practices.

Additional Readings

"Common Sense About the Common Core" by Alan H. Schoenfeld. September 21, 2014. A straightforward explanation of the Common Core and its future.

"Tying It All Together: Classroom Practices That Promote Mathematical Proficiency for All Students" by Jennifer M. Suh. October 2007. Published in Teaching Children Mathematics.

> A professor's practices which tie together multiple beliefs and good teaching practices into one, coherent stance on teaching mathematics.

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