Teaching Strategies for the High School Math Classroom

By

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Abstract

As technology, social interactions and diverse cultures grow; teachers are faced with the challenge of creating informative, relevant and interesting lesson plans for the current generation. While this thesis started out as a creation of two unique lesson plans, it turned into a journey through different teaching methods and the theories that back those methods. This thesis highlights two teaching strategies: Inquiry-Based Learning and the use of Real World examples. These methods can be applied to most middle school or high school mathematics classrooms. Two lessons are given as examples of these teaching methods. A Cryptology lesson introduces students to clock arithmetic and relates it to encrypting and decrypting codes. In addition, a Geometry lesson focuses on the relationships between similar and congruent triangles. Both lessons use the Inquiry-Based Learning method and Real World applications to actively engage students in what they are learning and thus, give them the opportunity to create meaningful learning experiences.

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Introduction

It should come as no surprise that teens of this generation are becoming more sedentary than any generation before. They have become accustomed to constant stimulation from the high amounts of electronic entertainment that they engage with on a daily basis. It is believed that, because they are spending their time outside of school in this way, the amount of students diagnosed with Attention Deficit Disorder or Attention Deficit Hyperactivity Disorder is on the rise. Thus, students are finding it harder and harder to sit through a traditional classroom lecture. According to Sue Freeman Culverhouse, studies show that the average attention span of a student is directly correlated to their age: two minutes, plus one minute per year. This means that, on average, students entering middle school have a 13-minute attention span, and by the time they graduate from high school their attention span is only 20 minutes. This information suggests that most students cannot be effectively taught by a teacher who lectures in front of a classroom for more than 15-20 minutes. Students need to be engaged in what they are learning. This involvement may mean walking around the room, having them present their findings to the class, or having them discuss a certain problem in groups. However, student involvement does not have to be displayed by a physical movement. It can also mean having students participate in what they are learning by coming up with their own practice problems, or filling in a worksheet designed to review a recent lecture. The possibilities are numerous. What matters is that students are staying stimulated and involved in what they are learning.

As technology, social interactions and students' expectations change and evolve from year to year, teachers are faced with the challenge of creating applicable, interesting and involved lesson plans that students of the current generation will respond to. Teachers are constantly striving to come up with new, effective teaching methods, while at the same time being faced with endless constraints. Oregon recently adopted the Common Core State Standards (CCSS). These standards outline what students should be learning in mathematics, language arts and social studies each year. Teachers who want to include a lesson plan that teaches curriculum outside of the CCSS are permitted, but there is often little or no time to do so. Thus, educators are finding that there is not enough time in their lessons to be creative while still meeting CCSS requirements.

There are many teaching methods that can help rectify the previously discussed problems. Two of which are "Inquiry-Based Learning" and intigrating "Real-World Problems". Each of these methods are not lessons in themselves, rather, they are approaches that can enhance any lesson plan. Inquiry-Based Learning (IBL) is a method in which the teacher acts as the facilitator and students are asked leading questions to help them come to their own conclusions about what they are learning. This allows students to be actively involved in what they are learning by discovering patterns and formulas that they can apply to future problems. The use of real-world problems are beneficial in that they allow students to see how math is applied in real life situations. It gives their assignments meaning, and engages them in something that has a purpose. Each of these methods can improve a pre-existing lesson plan by engaging students more actively in their learning process.

Inquiry-Based Learning

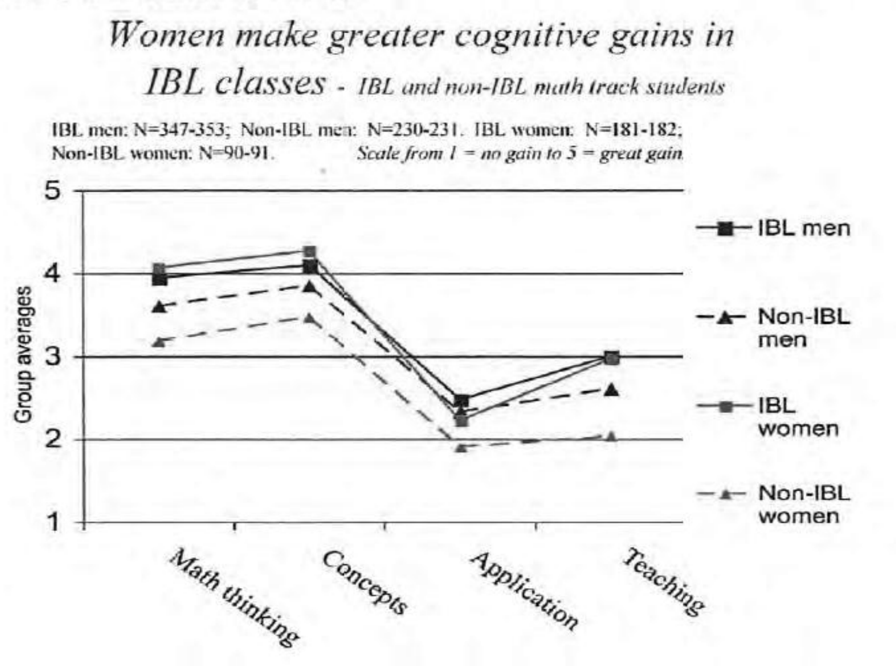
The general idea of Inquiry-Based Learning (IBL) is for the teacher to ask guided questions of the students, leading them to discover mathematical processes and theorems on their own. It has been argued that the method of "lecturing and listening" is not the ideal way for most students to learn. Instead, teachers should act as facilitators in their classrooms. By using the IBL method, students are given the freedom to come to their own conclusions. A report, issued by the National Research Council entitled *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* stated: "In reality no one can teach mathematics. Effective teachers are those who can stimulate students to *learn* mathematics... students learn mathematics well only when they *construct* their own mathematical understanding." (pg 58) Through their inquiry and construction, students will gain a more meaningful learning experience.

Inquiry-based learning is not a new concept. The earliest known documentation of its use dates back to the 18th century when Warren Colburn published multiple texts emphasizing the fact that students should be inventing their own computational procedures and arithmetic. In 1819 educator Samuel Goodrich asserted that "teaching arithmetic by rote actually prevented children from understanding arithmetic and… they should discover rules by manipulating tangible objects." In the article *Everybody Counts*, advocate Jean Piaget (1896-

1980) stated: “to understand is to invent.” Over the years many prominent figures have advanced the use of IBL including Maria Montessori (1870-1952) and John Dewey (1859-1952) who both argued that students should play a larger role in “directing their own studies and constructing their own understandings.” Montessori and Dewey were not focused solely on mathematics education. Still, it is clear that their support encouraged a wider backing of the IBL method in many subjects, including mathematics. Since Colburn’s initial promotion, the popularity of this method has come and gone in waves. However, it cannot be ignored that the IBL method has merits that are being recognized more and more by teachers of this generation.

It is obvious that IBL has had a large following historically, but some may still question whether it is an applicable and beneficial technique in today’s world of education. One of the simplest examples of IBL that is used in nearly every student’s early education is when they are learning addition and subtraction. Often times teachers will not tell the students that 2+2=4, but instead will give them two objects, then two more objects and then ask how many objects they have. By moving physical objects, as discussed in this example, students are able to better “discover” that 2+2=4 on their own. However, the benefits of the IBL method stretch far beyond basic arithmetic. In the 1920’s R.L. Moore began using this method in his college calculus classes at the University of Texas. Moore broke the traditional mold by not allowing his students to consult textbooks. Instead, during class he encouraged them to derive their own formulas through proofs done in a sequence that he prescribed. Here, students were creating their own learning, while still indirectly acting under Moore's guidance.

In 2010, a team of researchers led by Dr. Sandra Laursen of the University of Colorado at Boulder Ethnography and Evaluation Research unit gathered data collected at the 2010 Legacy of R. L. Moore Conference held in Austin, TX. This study showed that in nearly every area IBL students scored higher than those who were not taught with the IBL method. The results from the study are displayed in the following graph:



Though the objective of this study was to highlight a gender gap, it also shows that, no matter one's gender, in almost every category, IBL students out-scored non-IBL students.

Though there is a lot of evidence in support of the IBL method, it cannot always be implemented in a successful way. There is the possibility that students who discover their own ideas or theorems may come to incorrect conclusions. For this reason, teachers must stay involved in watching each step that the students take, and correct any errors that they may have along the way. Another challenge that teachers face when trying to implement an IBL method in their classrooms is when they are not always sure *how* to lead the students to a desired conclusion. Often teachers can ask the "right" questions that will build on the previous knowledge of their students. However, there are also times when teachers must do more to guide students in the right direction. For example, certain formulas used in a high school Algebra II class would be much too difficult for high school students to prove. Thus, it is necessary at times to "feed" students the tools that they will need to be able to continue in their learning. Teachers may also end up "feeding" students the information when they do not have enough time for all of the students to discover the information on their own. Depending on the difficulty of a concept, and the grade level of the students, IBL often takes more time to teach concepts than a more typical lecture setting. For this reason, it may not be practical to use the IBL method in a math class every day. Still, it can be extremely helpful when used sporadically or with certain topics.

Despite the challenges associated with the IBL method, many would argue that the benefits to implementing this method from time to time are still greater. Perhaps the greatest advantage to this method is that it can be used in all grade levels and in many subjects. Though this paper is limited to mathematics, the thinking skills that students exercise by going through an IBL mathematics lesson plan can be applied to other subject areas as well. An important part of IBL is to create inquisitive and self-motivated students inside and outside of the classroom. Because the teachers are not directly lecturing to the class, students must pull from their own background knowledge and the world around them in order to solve a given problem.

Applications of Inquiry Based Learning

This thesis presents two lesson plans that use IBL to some degree, one in Cryptology and the other is Geometry. In the Cryptology lesson, students are asked leading questions for the purpose of relating how to tell time with modular arithmetic. The modular arithmetic worksheet and the lesson plan adhere to a lesson in which students should be individually discovering how to encrypt and decrypt codes. There are terms, such as "*multiplicative inverse* under modular arithmetic", which the instructor will have to explain to the students. However, the students should be capable of relating this term to inverses in multiplication that they have learned about in the past, helping them to better understand the concept.

In the Geometry lesson students are given more guidance as there are many definitions that would be difficult for middle school students to derive. However, relationships and properties of those definitions are left for the students to discover. For example, students may be given a picture of two angles that are defined to be supplementary, but they must discover on their own what the relationship between the two angles is, meaning that the angle measures add to 180º. Students are also given the opportunity in this lesson to complete a proof, and discover that similar triangles have sides with equal ratios.

Both lessons use IBL to help students gain insight into the key concepts of the lessons. Whether it is definitions, theorems, or a process used to solve an equation, students are given the opportunity to play a vital role in their learning. In addition, the instructor is constantly guiding and facilitating what definitions and logistical reasoning the students are using in order to keep them from reaching the wrong conclusions.

Real World Problems

Most educators would agree that teaching mathematics must to go beyond the memorization of rules and theorems. Students must also be challenged to understand *what* they are using and *how* it can be applied to real world situations. Real world problems can take many forms and are not necessarily only connected with the subject of mathematics. These problems involve different concentrations of math, science, social studies, and even language arts.

The purposes of using real world problems are threefold: first, real world problems serve as a device to make mathematics come alive for students. Real world problems have the ability to capture students' attention in a way that most mathematical equations and theorems cannot. The second purpose of using real world problems is to give students the opportunity to relate what they are learning in the classroom with the world around them. Students who are able to relate their mathematical knowledge to real world situations are more apt to remember the methods that they used in future situations. Lastly, real world problems give students the chance to relate math to other subject areas such as physics, engineering, or economics. Through this, students are able to see that various subjects do not act in isolation. Furthermore, they discover that what they are learning has a purpose and function in the world outside of their classroom.

The implementation of real world problems can take many forms. To explain division to a fourth grade classroom a problem could be as simple as the example, “Johnny gets $3 a week for allowance and wants to buy a $35 toy. How many weeks of allowance will he have to save?”. In a high school calculus class a teacher could assign a homework assignment where students have to solve a story problem related to physics. Some problems require students to work on their own, while others require students to work in groups. Real world problems can be done at one’s own desk, or they may require students to move around the room or even venture outside of the classroom. Each real world problem more actively involves students in what they are learning rather than taking part in a teacher directed lecture.

Although there are many benefits to using real world problems in the classroom, teachers should also be cautious of implementing this teaching method. There are three main drawbacks associated with using real world problems. First, students may not extract the mathematical significance of the problem that they are solving. This hinders a students’ ability to apply the math behind one problem to multiple other situations. Another disadvantage of using real world problems is that teachers often assume that all students have the same background knowledge. This can particularly affect students from different cultural and socioeconomic backgrounds. Some students may be able to relate to a given problem better than others; some may not be able to relate to the problem at all. Consider the example of finding the probability of a certain seven card hand with a deck of cards. Most students will be familiar with playing cards, while there will be others who have never seen a deck. It is important for teachers not to assume that each one of their students has the same background knowledge. The last disadvantage of using real world problems is that they often take more time to explain and solve than a straightforward algebraic or graphical problem. Although some may argue that this time is well spent due to the benefits of understanding the real world significance of particular math concepts, teachers often run into a time shortage to teach their lessons and cannot afford to include many real world problems in their lesson plans.

Perhaps the largest concern regarding the use of real world problems in math classes revolves around the amount that teachers rely on the problems to teach mathematical concepts. In her article, "The Problem with Real-World Problems", Sarah Lubienski states that "as teachers, we cannot assume that a real-world problem will "carry the mathematics"... we must continuously assess what children are learning, including whether they understand the mathematics in such a way that they can transfer their understanding from one situation to another." It is evident that there are many pros and cons to the use of real world problems in a math classroom. This leaves us to question: *what is the best* *balance of using real world vs pure math problems to teach mathematics?* Lubienski warns that students may have "difficulty building deep mathematical understandings" if an integrated real world approach becomes the staple of mathematical instruction. However, the doubts behind the implementation of teaching methods that utilize real world problems are mostly overshadowed by their positive attributes. According to the article *Math in Daily Life*, "human beings didn't invent math concepts; we discovered them". Math is omnipresent. What better way to teach it than to harness the *real* examples that we find around us in the world everyday? Though this is just one stated opinion, it is one that is echoed throughout much of the mathematical teaching society.

The following lesson plans employ the use of real world problems to interest students in what they should be learning. The Cryptology lesson plan connects to the real world by teaching students how modular arithmetic can be used for encrypting and decrypting codes. Coding was a popular subject during WWII when the United States used cryptologists to decode encryptions, much like the ones that the students will be working with, in order to intercept German and Japanese messages. This real life application will no doubt leave students anxious to start making and breaking codes just like the cryptologists who were in the war.

The Geometry lesson plan initially gives students the opportunity to create similar triangles by “walking out” these triangles with their own steps. This allows students to create their own triangles in addition to writing them on paper. It also allows them to see commonalities in the similar triangles’ sides and angles. The story problems at the end of the lesson connect this math to the real world by directly involving students in the school around them. In fact, everything in the lesson builds up to being able to solve these problems. These problems act as the assessment for the lesson. Thus, students are left with the impression that real world applications play an important part in their learning.

Real world problems are extremely beneficial for any lesson plan because of their potential to play diverse roles. They can be incorporated easily with simple examples given by the teacher, or they can be tagged on to the end of a lesson to show students that the information they have learned has purpose in the real world. These problems are rewarding in that they can be tailored to connect with students depending on their individual interests or their grade level. Given that the instructor takes the time to get to know their students, and has the time to individualize some of the problems, students will be able to benefit greatly from these exercises. Additionally, it is not always necessary that the teacher have ready-made problems for each student. Teachers can at times ask the student to come up with their own real world problem. This ensures that students are actively engaged and invested in the concepts that they are learning. It can also act as a form of assessment for students at the end of a lesson.

Common Core State Standards

On October 28th, 2010, Oregon's State Board of Education adopted the Common Core State Standards (CCSS). These standards "represent k-12 learning expectations for students in English-language arts and Mathematics." These standards are meant to be relevant to the real world and to instill the knowledge that students will need in order to be successful in college and in future careers. In the past, each state has been responsible for their respective educational curriculum guidelines. Educators are beginning to recognize that schools from different states are not all adhering to the same standards, resulting in inequitable education for the students across state lines. The CCSS will help ensure that students from all states will be equally prepared to enter a college or job in any state in the U.S. For this reason, since June 2010, over 40 states have adopted the CCSS.

It is important to note that the Cryptology and Geometry lesson plans meet multiple standards outlined by the CCSS. The following is an explanation of the guidelines that are met and thus, the audiences that the lesson plans are targeting.

The Cryptology lesson is one that can be used in a variety of classes with students from a range of ages and math abilities. The background knowledge required of the students in order to participate in this lesson is that of introductory algebra. Students must be confident in doing basic arithmetic including adding, subtracting, multiplying and dividing. They should also have some exposure to simple algebraic equations and be able to use the order of operations correctly. Practice in solving for x in equations such as: $5+x=10$ and $4x+1=21$ will also come in handy throughout this unit.

Because of the algebra background that is needed, this lesson is accessible to most middle school students, and all high school students. According to Oregon's CCSS for Mathematics, students in seventh grade work on developing an understanding of operations and expressions with linear equations and students in the eighth grade students become more comfortable solving linear equations. In addition, they should be able to define, analyze and evaluate functions. By the time most students are in their second year of high school they have the required math experience to be able to easily comprehend the concepts of this lesson.

The average high school offers math content classes in Algebra I, Geometry, Algebra II, Trigonometry, Pre-Calculus and Calculus, though not necessarily in that order or by those names. This lesson would be most applicable to an Algebra I or Algebra II classroom because of its reliance on equations and abstract algebra concepts. However, if time allows, it could be an interesting unit in a Pre-Calculus course as well because it better develops the concepts of inverses, identities and modular arithmetic in ways that are not commonly used in a traditional high school curriculum.

The Geometry lesson is ideally written for an eighth grade classroom. In the seventh grade, students work with ratios and are exposed briefly to geometric terms and problems. According to Oregon's CCSS for math students, seventh graders should be able to "Draw, construct and describe geometrical figures and describe the relationships between them." This exposure prepares students to delve deeper into working with similar and congruent triangles in the eighth grade. Also in seventh grade, in order to prepare for working with congruency and similarity, students will "reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions" in order to become familiar with "the relationships between angles formed by intersecting lines". In the eighth grade students learn "ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems." Therefore, it is apparent that according to CCSS, a lesson on similar triangles would be most fitting in an 8th grade classroom.

Though the optimum time to introduce this lesson is in the eighth grade, it could also be implemented in a high school Geometry class. In order to make it more of a challenge for this level of students, the teacher should have the students walk through more of the lesson on their own. For example, eighth grade students will require a significant amount of teacher directives to get through the proof that there are 180º in a triangle, while high school students should be able to discover it on their own or by working with a partner. The vocabulary in the beginning of the lesson will be new to most eighth graders, but will be review for those in high school Geometry. However, since it has most likely been a few years since the students have applied their knowledge of this vocabulary, high school students may need some hints and prompting from the teacher or other classmates. The most significant difference that will arise from teaching a high school verses an eighth grade class is that the lesson will take much less time, as the older students should be able to pick up on patterns and be able to understand instructions at a quicker pace. In the following section, when time approximations are given for each part of the lesson, the times are more indicative of the pace of an eighth grade classroom.

Background Information for Teachers

Before engaging a classroom in either the Cryptology or Geometry lesson plans that will be outlined in the next section, it is important to discuss what information the teachers should be comfortable with. For the Cryptology lesson plan, an instructor should be familiar with modular arithmetic and the concepts of inverses and identities when working modulo 26. Modular arithmetic (also known as clock arithmetic) is "a system of arithmetic for integers where numbers "wrap around" a certain value- the *modulus*." The majority of the Cryptology lesson deals in modulo 26. This means that, though we may operate (add or multiply) numbers greater than 26, we divide by 26 and take the remainder to get our answer. Modular arithmetic allows us to align a number with each letter of the alphabet (A with 0, B with 1, C with 2... Z with 25) and add or multiply a given set of numbers to encrypt a message producing only numbers from the set 0 through 25.

In Cryptology, one can encrypt a message given a *shift* (additive) or *affine* (multiplicative) cipher, or code, with a somewhat basic knowledge of modular arithmetic. A shift cipher will take a code where A is paired with 0, B is paired with 1, C is paired with 2 etc, and move the letters up a given number. For instance, a shift of one will leave A paired with 1, B paired with 2, C paired with 3 etc. An affine cipher takes the same basic code, and multiplies each number. For instance, an affine cipher of 3 will leave A paired with 0, B paired with 3, C paired with 6 etc. In addition, in order to know how to encrypt and decrypt successfully one must know what it means for two numbers to be *relatively prime.* Numbers that are relatively prime have no common factors. In other words, their greatest common divisor is one. In this lesson, students will be able to see that in order for an affine cipher to produce a "good" encryption, i.e. an encryption where each number (0-25) is assigned exactly one letter, two numbers must have a "greatest common divisor" (gcd) of one. This will allow there to be a multiplicative inverse, making the code reasonable to decrypt. Consider the following example for when the gcd is not 1: say we use an affine cipher of 2 when working modulo 26, then A will stay paired with 0, but N, which was originally paired with 13, now becomes 26 and 26 is equal to 0 when operating modulo 26. This means that someone trying to decrypt the code would not be able to differentiate between an A and an N. Thus, 2 is an example of a "bad" cipher because the encrypted code does not have a one-to-one correspondence between the letters of the alphabet and the numbers 0-25.

The instructor must be comfortable with both additive and multiplicative *identities* and *inverses* when working modulo 26. An *identity* is defined as a number that has no effect on what it is operated with. For example the identity when operating under addition is zero, and the identity when operating under multiplication is one. By adding zero, or by multiplying by one, the original numbers that we start with will not be changed. *Inverses* are defined as numbers which, when operated (added or multiplied) with another number, will give the (additive or multiplicative) identity. Whereas the identity only depends on the operation being done, and not the original number being operated, inverses are dependent upon *what* number is being operated. Students may be familiar with additive inverses in that they are simply the negation of the original number i.e. 2 and -2. Students may also be familiar with multiplicative inverse referred to as the *reciprocal* of a number. For example, the multiplicative inverse of 2 is 1/2. However, in modular arithmetic we can only add and we can only deal in integers. Thus, inverses look a bit different. An additive inverse means that we must add on a certain amount of multiples of our modulus (in our case- 26) until we get back to our additive identity (zero). Similarly, a multiplicative inverse means multiplying the number we are given until we have a remainder equal to our multiplicative identity (one). For example, 17x23=391 and 391 has a remainder of 1 when divided by 26, thus 17x23 is congruent to 1 modulo 26 and 17 and 23 are inverses of each other.

Outline of Lessons

Cryptology:

In this lesson plan students will begin learning about modular arithmetic by first looking at the 12 hours on a clock. After being guided through some simple problems they will be given some more difficult situations where they will be asked to do more than simply count ahead x amount of hours. Students will then be asked to explain how it is that they came up with answers to questions such as: “What time will it be in 100 hours?” Students should then be introduced to the term *modular arithmetic*. This piece of the lesson should take about 15 minutes of class time.

The class can then move on to aligning the alphabet with the numbers 0-25 and begin writing codes. They will soon learn that a shift of zero is an easy code to crack and will be asked how one could make a more difficult code. They will use their new knowledge of modular addition to *shift* which letters are aligned with which numbers. For example, a shift of one might move the number one from being aligned with A, to being aligned with B. Students will then practice encrypting and decrypting codes with a given shift cipher. They will be given the opportunity to split into pairs and encode their own messages for another pair in the class to decode. This section of the lesson, including the class activity, will take about 20 minutes.

The students should then be asked to consider how they would decode their messages if they were only allowed to add, and not subtract from the code they are given. After completing a few examples, and with guidance from the teacher, students will begin to understand the concepts of additive inverses and identities for modular arithmetic. Depending on the grade level of the audience, this may take anywhere from 2-10 minutes.

Next, the teacher should explain the concept of an *Affine Cipher*. Students will begin by only looking at the first 10 letters of the alphabet. They will then use an affine cipher of three and decode a message. Next they will repeat this process with a affine cipher of two. After each decryption they are asked whether that number is a smart number to encrypt with. Students will come to understand that two is not a smart number because some of the letters become paired up with the same number making it impossible to decode. They will then discover, through trial and error, which numbers do or do not work with the first 10 letters of the alphabet. With perhaps a small amount of class discussion, students will be able to come up with the affine ciphers that will “work” when we are using all 26 letters of the alphabet. This portion of the lesson will take about 10 minutes.

Much like the activity with shift ciphers, students will break into pairs and encrypt a short message for another pair of students. Students will then be asked to consider how they would decode a message if they were only able to multiply the code they were given. This will lead them to understand the concept of multiplicative identities and inverses. This part should take 15-20 minutes of class time. In the last few minutes of class the teacher should review everything that has been learned during the day, or during the unit. Perhaps as a homework assignment students could write more codes for their friends to decode. If there are more advanced students in the class, challenge them with creating a code that uses both a shift and an affine cipher. Depending on the amount of time available during one period of class, this lesson may be done in one 70-85 minute day, or two 35-45 minute days.

Geometry:

In this lesson plan students will begin by being introduced (or re-introduced) to important geometry vocabulary. In order to do this, instructors will begin by passing out a vocabulary/relationships sheet and a protractor to each student. Ask the students to write down the definitions to the words that they recognize. For some of the words, such as “corresponding angles” and “vertical angles”, students are given a picture. Using their protractors students will be able to notice some relationships between certain angles. Students may want to use the back of their paper, or some scratch paper to draw multiple vertical angles etc. in order to realize that the opposite angles will always be congruent. They can conjecture this from their measurements, or they can prove it with what they know about supplementary angles. After the students have come up with their own definitions of the vocabulary and worked with the relationships, the class as a whole should discuss what they wrote down, and come to a consensus (along with the teacher) on what the definitions and relationships should be. Depending on the grade level of the students, this part of the lesson will take about 10 minutes.

The instructor should next pass out three note cards to each student. They should tell the students to use their protractors to draw three triangles, one on each note card. Make sure to tell the students to make each triangle a unique shape and/or size; students should also label each vertex (abc, def and ghi). Next, have the students measure and record each angle of each triangle that they have. Upon collecting this data from the class, the instructor should ask the students what they think the angle sums of a triangle are, and how convinced they are of this conclusion. Give the students 5-10 minutes to try and prove that the angle sums of a triangle sum to 180º with a mathematical proof. Remind them that they will have to use some of the vocabulary that was recently discussed. Unless the majority of the class is able to come up with a mathematical proof, the teacher should plan on either presenting the proof (attached to the lesson plan), having a student present their proof, or having a pair of students present what they have. This section of the lesson will take 15-20 minutes.

Next, hand out the worksheet with 10 different triangles on it. Have the students group the triangles into three groups. The goal is for them to make three groups: similar triangles, congruent triangles or neither. As a class, discuss how everyone grouped their triangles. Introduce the terms *similar* and *congruent* during the class discussion. Have a student give you (the teacher) the angle measures of one of their triangles. Replicate a triangle on the board with the angles given. Note that your triangle is much larger than theirs, but since it has the same angle measures it is considered to be similar to the student's triangle. This portion of the lesson will take about 10 minutes.

By measuring the sides of their similar triangles, students will discover the relationship between the ratios of the sides. Depending on the grade level, it may take a short class discussion to come to this conclusion. Students are then asked to "walk out" their triangles outside, or in a large open area. They should use their feet as units of measurement to create triangles similar to the ones on their note cards. Students should use their new knowledge about ratios of sides to make sure that the sides of the triangles that they walk out have the same ratios of the ones on their note cards. They may begin by walking out x feet for each side that is x inches or x cm, but challenge them to create different sides that will still have the same ratios. If this activity is done outside, chalk may come in handy to draw the sides or mark the vertices. If done inside, masking tape is a good substitute. From answering the questions, to completing the “walk it out” activity, this section of the lesson will take between 20 and 30 minutes.

As a final activity, students will be split into groups of 3-4 to go around the building and apply their new knowledge to some real world problems plaguing the faculty. Each group should be given a different problem to solve. They may have to find angle measures, or ratios, or both to solve the problems. Note that students will be more interested in what they are doing if the problems are tailored to their school. Changing the teachers’ names to ones that the students are familiar with is a good way to get them more involved in solving the problems.

This last piece of the lesson could take anywhere from 10-20 minutes depending on how much freedom the teacher wants to grant the students. They may want to send the students out to collect data for five minutes, and then return to the classroom to fill in the math before a final class discussion, or they may be given 15 minutes to go out and find the answer and then come back and share with the class. This will depend on the grade level of the class, and the supervision that they may need outside of the classroom. This lesson will need to be split up into two or three days depending on the length of the class period. It should not take more than two hours of total class time, but will most likely take more than one and a half hours. This will also depend on whether it is being implemented in a high school or middle school setting. Note that if it is split between more days, teachers may need to account for the time that it will take to review what was done in previous days before they can move forward in the lesson.

Cryptology Lesson-Plan

Cryptology/Mathematics
8^{th} grade to Pre-Calculus
70-85 minutes

Goals: Introduce and use modular arithmetic to encrypt and decrypt codes. Understand the roles of inverses and identities in *shift* (additive) codes and *Affine* (multiplicative) codes.

Objectives: Students will become familiar with modular arithmetic. Students will be able to create and solve codes between themselves and their classmates. Students will understand the uses of additive/multiplicative inverses and identities. Students will gain deep knowledge and confidence as they discover these methods through an *inquiry-based learning* system.

Prerequisites: Students should be familiar with the alphabet and with telling time on an analog clock. Students should also be comfortable with addition, subtraction, multiplication and division of whole numbers.

Materials:

Worksheets for each student
Calculator for each student

Procedures:

Start by handing out the worksheet. Have students answer the first five questions.

Explain that the "clock-calculations" they are doing are equivalent to working "*mod12*".

Switch over to aligning the alphabet with 26 numbers (0-25).

Have the students decode one or two simple messages with no shift.

Explain that this is the same as a shift of zero, thus zero is the *additive identity*:

0 (or 26) is the additive identity of x(mod26).

Ask them to write a definition of what an additive identity is.

Ask them to relate this to the additive identities that they are familiar with.

6.Shift +3 (*Caesar cipher*). Have the students figure this out and see if they can relate it to the *modular arithmetic* that they were dealing with earlier with the clock.

7. Practice decoding a message with a shift of 3.
8. Break the students into pairs.
9. Have each duo come up with a simple message and number to shift their alphabet by.
10. Have each pair trade with another pair and decode each other's messages.
11. Inquire as to how the students decoded their message, aka the *cipher text*. Have the students tell you what they did, keep a mental tally of whether they did: (1) or (2).
    1. Did they count back that many in the alphabet?
    2. Did they count forward instead?
12. Tell them to suppose that they are only able to count forward in the alphabet, as is true for modular arithmetic. Have them figure out how many letters they would have to count forward in order to decode the message they were given.
    1. Give them a shift of 24 and inquire as to how they would decode a simple message.
    2. Hopefully students' responses will lead to a general consensus that if we encrypt with a shift of 24, it will take too much time. Instead we have to look at how many letters we would have to count forward to get back to the *plain text* (note that this is the number that brings us back to our additive identity).
    3. We see that if we count forward 2 every number is paired with the letter that it was paired with A with 0, B with 1 etc. This is called the *plain text*. Therefore, 2 and 24 are additive inverses under mod26.
13. Explain to them that this number is the *additive inverse*:
    1. Have them relate this to *additive inverses* that they are familiar with.
    2. Ask them to write a definition of additive inverse.
14.Repeat the group activity for an affine cipher.
15. First have them multiply their plain text by one.
    1. Ask them to explain why this is not a good cipher. (Can they relate it to 0 as the additive identity?).
    2. Lead them into telling you that 1 is the *multiplicative identity.*
    3. Ask them to relate this to *multiplicative identities* that they are more familiar to.
    4. Have them write down a definition of multiplicative identity.
16. Have them try working mod10 with just the first 10 letters of the alphabet and figure out which affine ciphers will or will not work. (1, 3, 7 and 9 work, 2,5,4,6 and 8 do not).
17. May have to have them work with mod15 and figure out which affine ciphers will and will not work.

    1. The goal here is to have them say either that the cipher and modulus must have a greatest common divisor (gcd) of 1 or, the cipher number cannot divide the mod number...

18. Make sure they are aware of what numbers will work when we are dealing with mod26.
19. Have them break into pairs and make a simple message and write down what affine cipher they are using.
20. Have the students trade with another duo and decode the cipher text.
21. Tell them to suppose that they are only able multiply the alphabet and have them figure out what they would have to multiply by in order to decode the message they were given.
    1. Example: If we encrypt with an affine cipher of 9, what would we have to multiply by to get back to our *plain text* (our multiplicative identity).
22. See if they can relate this to the *additive inverse* that we already talked about and tell you that this number is the *multiplicative inverse*:
    1. Re-state "3 is the *multiplicative inverse* of 9mod26".
    2. Ask the students to relate this to multiplicative inverses that they are more familiar with.
    3. Ask them to write a definition of multiplicative inverse.
23. Use the included multiplication table Modulo 26 to assist students in finding inverses.

Meeting Varying Needs: This lesson meets the needs of a variety of learners. It allows interpersonal students to interact with other as they trade notes with others in the class and work in pairs to decode messages. It also allows mathematical learners to thrive in applying their number-smarts to cryptology. Those who are spatial learners (picture smart) will be able to fill in the charts and rely on them as a visual each time they choose a new code and work to code or decode it.

Formative Assessment: The assessment is built into the lesson. Students must immediately apply their knowledge of modular arithmetic with shift or affine ciphers in order to create a code and encode a message for their classmates. They must also be able to use what they have learned about identities and inverses to correctly decode the messages they are given.

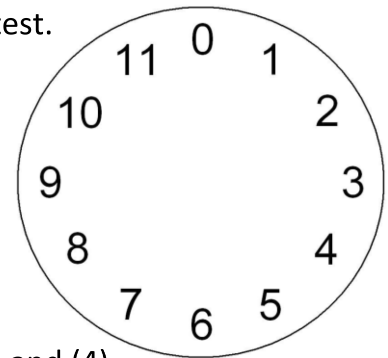
Cryptology Worksheet:

1) It is 8:00 and I have a math test in 4 hours. What time is my test at?

2) I went to bed 9 hours ago, what time did I go to bed?

3) In 29 hours I get to see what I got on my math test.

What time will this be?



4) There is a party in 100 hours.

What time is the party at?

5) Explain how you discovered your answer for (3) and (4).

Now, imagine that you are using a 26-hour clock.

Shift Ciphers

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

6) Decode the message: 7 0 21 8 13 6 5 20 13 22 8 19 7 12 0 19 7 !

Now try with a shift of 3 (write in the new numbers):

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
3					8					13						19						25			

7) Decode the message: 9 7 22 22 11 16 9 3 4 11 22 10 3 20 6 7 20…

8) What if we only used a shift of 2?

Decode the following message: 10 20 21 9 10 20 6 2 19 5 19?

Now pair up, pick your own shift and create a message for some of your classmates.

9) Explain how you decoded the last message you were given:

10) Is a shift of 0 a good cipher? Why?

11) An *Additive Inverse* is:

12) An *Additive Identity* is:

13) Give some examples of *Additive Identities* that you are familiar with:

14) How is *Additive Inverse* related to the *Additive Identity*?

15) What are some *Additive Inverses* that you are familiar with?

16) What is the additive inverse if you shift by 23?

17) What is the additive inverse if you shift by 7?

Affine Ciphers

What if we were to multiply our plain text by a number instead of adding a number to it? This is the difference between and *affine* and a *shift* cipher.

Note that the following is the cipher you would get if you multiplied everything by one.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

To begin we will simplify things by working with just the first 10 letters of the alphabet:

A	B	C	D	E	F	G	H	I	J
0	1	2	3	4	5	6	7	8	9

18) What is our cipher text if we multiply everything by 2 and reduce modulo 10 (just as we reduced modulo 12 or modulo 26 in the past)?

A	B	C	D	E	F	G	H	I	J

19) Decode the message: 2 0 6

20) Is this a smart number to encrypt with? Why or why not?

21) What if we multiply everything by 3?

A	B	C	D	E	F	G	H	I	J

22) Decode the message: 1 4 8 1 0 6 2

23) Is this a smart number to encrypt with? Why or why not?

24) Test what numbers work and what numbers do or do not work when we are dealing with mod10:

25) What is the relationship that you see between 10 and "good" affine ciphers that you found? What is the relationship between 10 and the "bad" affine ciphers that you found?

26)Similarly, test which affine ciphers will work when we are dealing with the first 15 letters of the alphabet:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

27) Can you see a relationship between 15 and the “good” affine ciphers that you found and the “bad” affine ciphers that you found?

28) In general, what can we say about the relationship between the modulus and the “good” ciphers? What about “bad” ciphers?

29) Test out and write down what numbers would be good ciphers when we are dealing with the full alphabet (mod26). Try affine ciphers of 2, 3, 4 and 5 to get started.

Now pair up, pick your own affine cipher and create a message for some of your classmates.

30) Explain how you decoded the message you were given:

31) A *Multiplicative Inverse* is:

32) A *Multiplicative Identity* is:

33) What is the relationship between the *Multiplicative Inverse* and the *Multiplicative Identity?*

34) What is the multiplicative inverse if you multiply by 4?

35) What is the multiplicative inverse if you multiply by 7?

Congratulations! You are now a Cryptologist!

Multiplication table Modulo 26

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	0	2	4	6	8	10	12	14	16	18	20	22	24	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	1	4	7	10	13	16	19	22	25	2	5	8	11	14	17	20	23
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5	0	5	10	15	20	25	4	9	14	19	24	3	8	13	18	23	2	7	12	17	22	1	6	11	16	21
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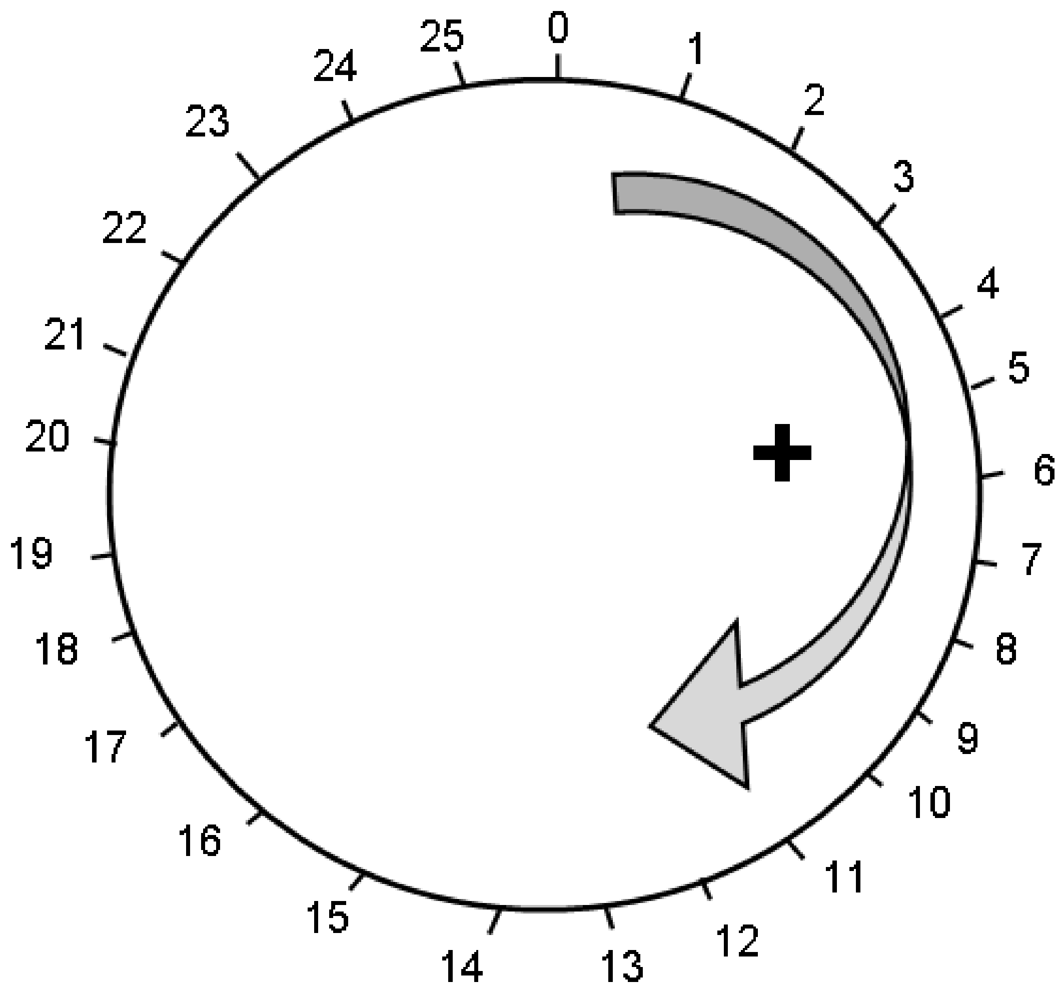
25	0	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Clock Arithmetic: Modulo 26



Cryptology Lesson Reflection

I had the opportunity to present my Cryptology lesson to the MTH 212 class taught by Cheryl Beaver. Because this class is meant for students who are pursuing a degree in elementary education, I expected a responsive and energized group of students who would be willing to work my unfamiliar IBL teaching methods. About half of the class met these expectations, while the other half lacked an enthusiasm for the subject, but begrudgingly followed along. Despite the many hours that I spent editing and refining this lesson, a lot of the prompts I planned on giving were quickly forgotten when I began working with the class. Some of this was due to the fact that the class caught on more quickly than I had anticipated and some was because I became caught up in the moment and forgot about the prompts altogether.

There were two steep learning curves that I had to overcome as I presented this lesson. The first came in trying to get the students to be as enthusiastic about the lesson as I was. I realize now that this would be unrealistic for most lessons, and that I should not let the students' lack of energy deter my own. A large reason that students were not as eager to participate at first was because I was a visiting teacher, and the class was at 8:00 in the morning. I believe that if I had prefaced the lesson with more background on the historical significance of cryptology more students would have been interested in the lesson from the beginning.

The second “issue” that I encountered was that some students were anxious to move ahead of the rest of the class. Throughout the hour, I found myself continuously telling students to slow down, or only do a certain number of steps before pausing for further instruction. Looking back I can see how wrong I was in my attempts to hold students back. In fact, one of the benefits of using the IBL method is that it allows students to work at their own pace. Students who are able to easily move through the lesson will not be held back by students who need more time to work through the steps. In addition, students who take more time will not be forced to move past concepts that they do not yet firmly understand.

Most of the students seemed to enjoy the lesson and the challenges that it presented. There were certainly things that I would explain differently next time. I would be sure to add real world (WWII) references to make the lessons more fascinating and applicable and I would allow students to work at their own pace. Once I account for these changes, I am sure that this lesson will engage a greater amount of students in a more meaningful learning experience.

Geometry Lesson-Plan:

Similar Triangles/ Mathematics
8th grade
90-120 minutes

Goals: Oregon's Core Content State Standards (CCSS) state that in the 7th grade students will be able to "draw, construct and describe geometrical figures and describe the relationship between them" in order to prepare them to work with similar and congruent triangles in the eighth grade. CCSS also states that in the eighth grade students will use "ideas about congruence and similarity to describe and analyze two-dimensional figures and solve problems."

Objectives: Students will become familiar with geometric vocabulary. They will learn the basics of how to put together a mathematical proof dealing with the degrees of a triangle. Upon learning about similar and congruent triangles, they will be asked to apply their knowledge to the world around them as they look at real-world examples.

Prerequisites: Students should be familiar with basic geometric terms such as *line, angle* and *triangle.* Students should be comfortable adding and subtracting three digit numbers. They should also understand the concept of ratios and using fractions to find and compare them.

Materials:

- Worksheet for each student
- 3 Note cards for each student
- Chalk or string (~25 feet per group)
- Protractors for each student
- Ruler for each student

Procedures:

1) Pass out a worksheet and protractor to each student and explain to them how to use the protractors.
2) Introduce vocabulary and notation and have students fill in their worksheet.
3) Pass out 3 (colorful) note cards to each student and a ruler.
4) Have the students use the protractor to draw three triangles, one on each note card.

5) Tell them to make the triangles however they want, but they must all look different. Make sure that they label each *vertex* (ABC etc).

6) Have students measure each of the angles of their triangles.

7) Have students follow the worksheet steps (3)-(4). Ask: “Have we *proven* that the three angles of every triangle add to equal 180º?”

8) Give students time to prove that the sum of any triangle’s angles will be 180º. Remind them that they will have to use some of their new vocabulary/propositions.

9) Guide the students in a proof.

10) Introduce *Similar Triangles vs. Congruent Triangles* by having students group the triangles they see at the front of the room. The teacher should draw on the board or project on a screen nine triangles: three congruent triangles, four similar triangles and two outliers. Tell the students that they can put them in three groups. This will involve a lot of class participation.

11) Have a student give you (the teacher) the angle measurements of one of their triangles. Draw a triangle on the board that has the angles they give you. Note that your triangle is much larger than theirs. This means that they are *Similar Triangles*. Note that they are not *Congruent Triangles*, because they are not the same exact triangle.

12) Have the students fill in steps (7)-(9) of their worksheet.
(The goal here is for the students to deduct that the ratios of the sides are the same.)

13) Have the students go outside and create triangles either with chalk or by laying out string. These triangles must have sides that are the same ratio as the triangles they have drawn on their paper.

14) Each student should use his/her foot as a unit of measurement.

15) Have each student show that their triangle is similar to the one on their paper and explain why.

16) Have the students complete the final story problems by splitting the class into 4 groups and assigning a problem to each group.

17) Once the students come back to the class with the answer, have them present how they found the answer by drawing their similar triangle on the board and giving the ratios they found.

18) Wrap-up by summarizing the important definitions and theorems that were used in the day’s lesson. Also lead a class discussion on each group’s experience around the school.

Meeting Varying Needs: This lesson takes into consideration many types of learning styles. Students who are “logic-smart” will excel with the proofs and in computing the ratios. Students who learn better by moving around will understand concepts through “walking out” their triangles. Interpersonal students will learn in their groups as they explore the triangles in the world outside of the classroom. Students who are visual learners or English language learners will learn concepts as they draw triangles and compare their shapes and sizes.

Formative Assessment: The assessment for this lesson is in the form of a student inquiry into the real world as students venture around their school. Students must have a firm enough grasp on the content that they have learned in class in order to apply their knowledge to real-world situations. They must also be able to internalize the information they gather, and the computations they make in order to explain the solution to their real world problem to their classmates in a clear manner.

Geometry Worksheet:

Vocabulary: Given the pictures, write out what you think the definition is in each box. In most cases you just need to state the relationship between the angles label "A,B" or "1,2,3,4" you may want to use your protractor to measure some of the angles.

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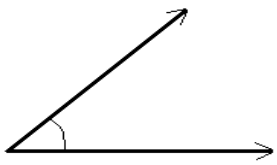
<td>Angle:
</td>
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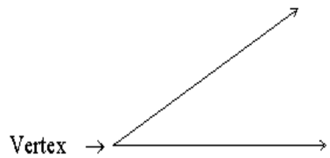</td>
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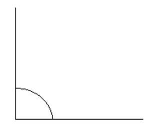</td>
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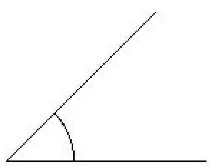</td>
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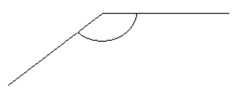</td>
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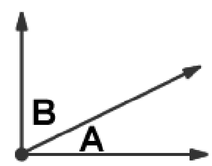</td>
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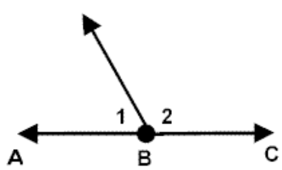</td>
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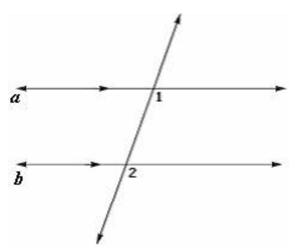</td>
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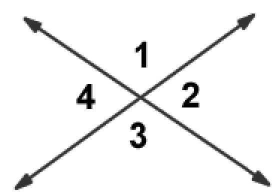</td>
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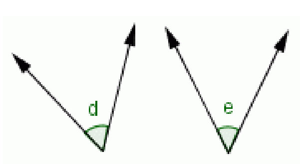</td>
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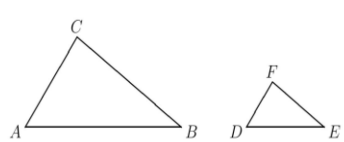</td>
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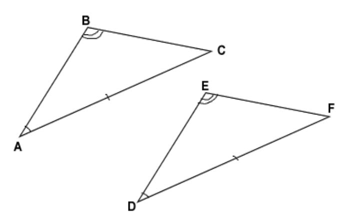</td>
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1. Use your protractor to discover the angle measurements of your triangles:

ΔABC:

Angle A=
Angle B=
Angle C=

ΔDEF:

Angle D=
Angle E=
Angle F=

ΔGHI:

Angle G=
Angle H=
Angle I=

2. Add up the three angles for each of your triangles:

Angles of ΔABC add to:

Angles of ΔDEF add to:

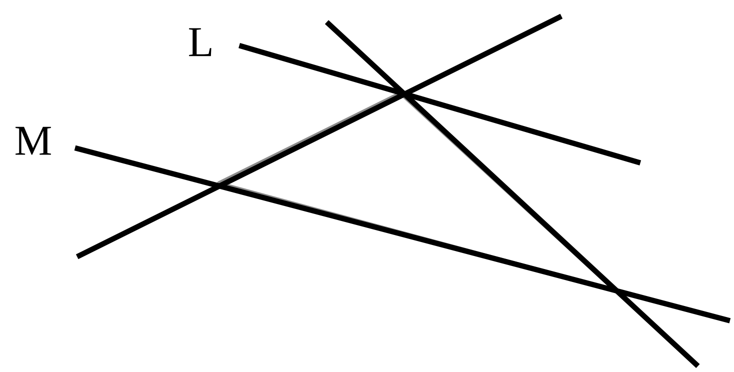
Angles of ΔGHI add to:

3. Can you draw any conclusions from the three calculations you just made? Do you see any similarities between the three triangles?

4. What is the conclusion that the class has come up with upon compiling data?

5. Prove using the figure below that the interior angles of all triangles will add to be ____º.

[Hint: You may need to compare *vertical angles, supplementary angles* and *corresponding angles* in your proof. Also note that lines L and M are parallel.]



6. In the space below make three smaller triangles that are similar to the ones on your note cards. Be sure to use your protractor to get the angles correct!

ΔABC ΔDEF ΔGHI

7. Using your ruler, measure the sides of all 6 of the triangles you have drawn and record in the space below:

8. Do you see any similarities between the sides of your first drawn triangles and the sides of their respective similar triangles?

9. From our observations we can guess that in addition to similar triangles having congruent angles, the sides also have the same ____________.

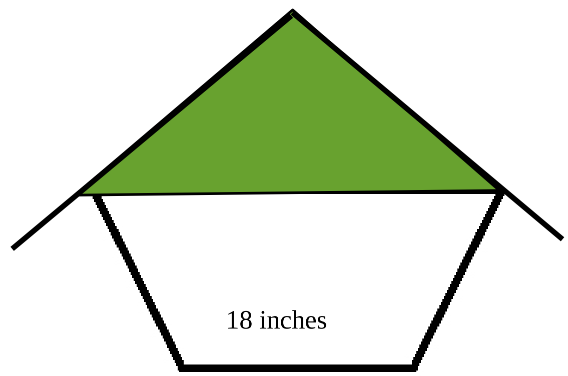
10. Take one of your triangles from earlier and “walk it out” outside attempting to make a much larger triangle that is *similar* to the one on your paper.

11. What are the measures of the sides of your triangle that you “walked out” (these numbers will be in your own unique unit.

12. How do you know that the triangle you “walked out” is similar to the one you drew on your paper earlier?

Now that we have a better understanding of similar triangles, lets use this information to solve some problems around the school.

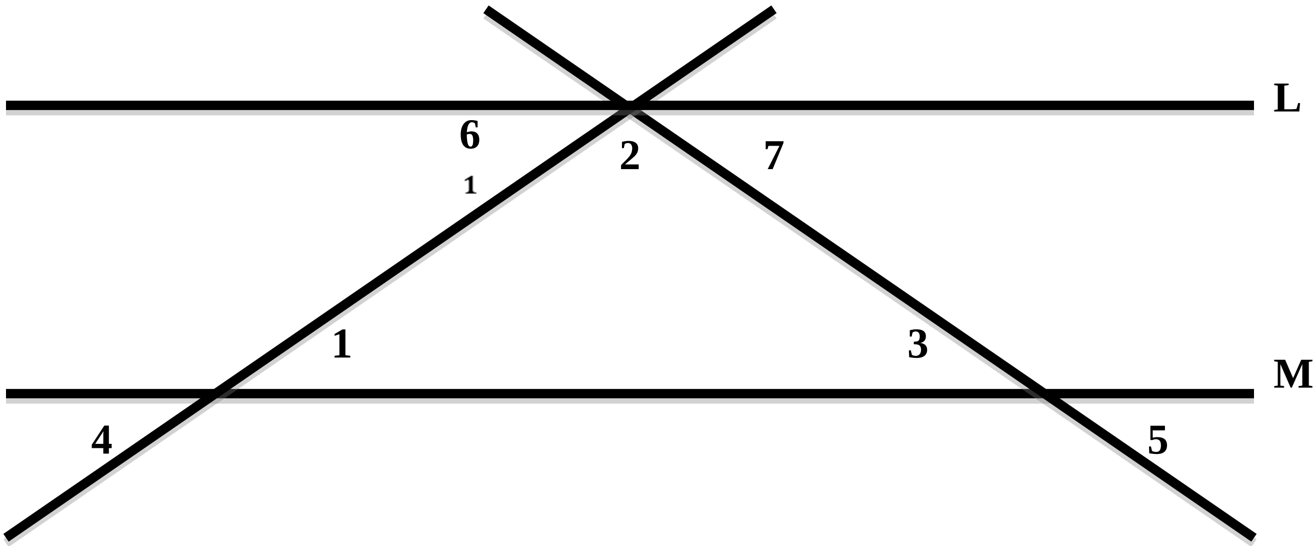
13. Mr. xxx, the science teacher, wants to put a new fish tank in his classroom. The tank is shaped like a hexagon, cut in half (as drawn below). He wants to squeeze it into a corner of his room where two walls come together at an 80º angle. If the side of the fish tank is 18 inches, how much wall space will he lose on either side of the corner of his room? Can you discover what angle his fish tank will intersect the wall at?



14. Mrs. xxx, the art teacher wants to hang some of her student’s artwork. She plans on hanging a line from one corner of her classroom to halfway across an opposite wall. Assume that the walls adjacent to her starting corner intersect at 90º and are each 28 feet long. The walls opposite her starting point are 30 feet long. How long will her string need to be? Use your protractor and a similar triangle that you have drawn to discover the angles of the triangle that she creates.

15. Mr. xxx, the PE teacher wants to roll tennis balls into the supply closet and needs to know at what angle he should do this. He is standing 30 feet down one side of the gym, and the length of the gym from him the wall he is standing against, to the door of the supply closet is 80 feet. At what angle does Mr. xxx need to roll the balls at, and how far is it from him to the door?

Proof: 180˚ in a Triangle



Given: Lines L and M are parallel.

Steps:	**Justifications:**
1. $\angle 1 = \angle 4$	Vertical Angles
2. $\angle 3 = \angle 5$	Vertical Angles
3. $\angle 4 = \angle 6$	Corresponding Angles
4. $\angle 5 = \angle 7$	Corresponding Angles
5. $\angle 6 + \angle 2 + \angle 7 = 180^\circ$	Supplementary Angles
6. $\angle 4 + \angle 2 + \angle 7 = 180^\circ$	Substitute using steps 3 and 4
7. $\angle 1 + \angle 2 + \angle 3 = 180^\circ$	Substitute using steps 1 and 2

Geometry Lesson Reflection

I presented my Geometry lesson to the MTH 213 class taught by Ron Weibe. This is the third of three math classes required for students who are planning on becoming elementary school teachers. As was true for the MTH 212 class, these students were also less enthusiastic about the lesson than I would have hoped. However, every one of the students was at least physically engaged in the lesson, and it was obvious that they enjoyed working with the other students in the class.

The biggest issue that I came up against while presenting this lesson plan was time constraints. I did not think that it would take so long for students to measure each of their angles. The work was more tedious than I realized, and students took great care in finding as precise measurements as possible. The fact that the triangles were drawn on note cards also made it difficult for students to measure the angles with their protractor. Since all of the students already knew that the angles of a triangle sum to 180º, they would "conveniently" write down what their third angle would have to measure in order to obtain this result. This method by-passed human error, but it did not force students to practice their measuring skills as much.

I was only able to get through about half of the lesson that I had originally planned on presenting to the class. Just as with the MTH 212 class, some students were ready to move ahead of the rest of the class, and this time I let them. Of the students who were able to move quickly through the first part of the lesson, some of them were able to correctly complete the proof that there are 180º in a triangle. The students who completed the proof were then able to share with their classmates what they did. I believe that students gain a deeper understanding when they are given the opportunity to explain their reasoning to their peers. Additionally, I believe that commonly students are more responsive to their peers than they are to a teacher who they are forced to listen to day after day.

As a whole, this class was more responsive than the MTH 212 class that I taught the Cryptology lesson to. In the future I will be sure to allow much more time for the completion of each step. I will also encourage them to measure all of their angles carefully with a protractor, and not just solve them algebraically. I understand that the smaller the angles, the higher chance there will be human error. However, I hope that with a large enough group we will be able to collect enough angle sums to find a class average close to 180º. Lastly, I will allow, and even encourage, students to work ahead if they feel that they understand each step. This will accommodate most levels of learners, and will therefore provide them with a more positive and empowering learning experience.

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